Meet 1

EVENT 1: Algebra II – Quadratic Equations & Functions

Notes: (1) If more than one letter is used in the problem, indicate which letter is the variable to be solved and which letters are constants.
(2) All equations must lead to a quadratic equation
(3) Domain and range should be expressed in set notation or interval notation. Suggestion: indicate set notation on the answer blank.
(4) Materials from OML's official reference for Algebra II

Include: (1) Real solutions
(2) Sum and product of roots
(3) Discriminant

Exclude: (1) Non-real solutions

Sample Problems:

A-1. Find the maximum value of $f(x) = -x^2 + 2x - 3$.
Answer: -2

A-2. Find the range of $f(x) = -x^2 + 2x - 3$.
Answer: $\{y \mid y \le -2\}$

A-3: Solve for x: $(x-4)^2 - 5(x-4) + 6 = 0$
Answer: 6 , 7

B-1. If one root of $2t^2 + 16t + c = 0$ is $-4 + \sqrt{7}$, find the value of c.
Answer: 18

B-2. Determine k such that the zeroes of $f(x) = 3x^2 + 6x + k$ are real.
Answer: $k \le 3$

C-1. If (2, 3) is the vertex of the graph of a quadratic function $f(x)$ such that $f(4) = 1$, find $f(x)$ in the form $f(x) = ax^2 + bx + c$.
Answer: $f(x) = -\frac{1}{2}x^2 + 2x + 1$

C-2. The sum of two numbers is 10. Find the numbers if the sum of their squares is to be as small as possible.
Answer: 5 , 5

Meet 1

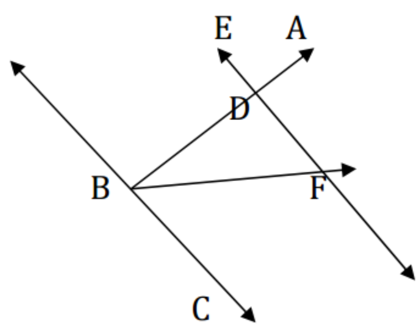
EVENT 2: Geometry – Angles in a Plane

Include: (1) Parallels and perpendiculars
(2) Complements and supplements
(3) Basic facts of triangles

Exclude: (1) Quadrilateral relationships
(2) Circles
(3) Area
(4) Similarity

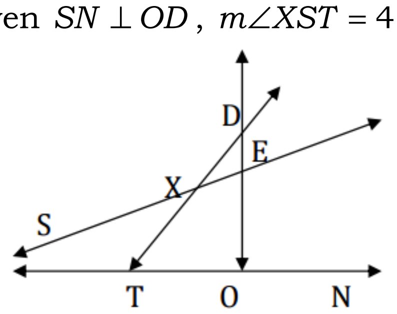
Sample Problems:

A. Given $\overleftrightarrow{EF} \parallel \overleftrightarrow{BC}$, $\overrightarrow{BF}$ bisects $\angle DBC$, and $m\angle DFB = 25°$. Find $m\angle EDB$.



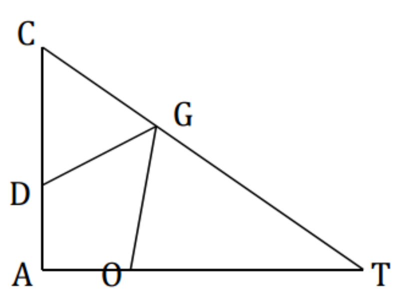
Answer: 50°

B. Given $\overline{SN} \perp \overline{OD}$, $m\angle XST = 45°$, and $m\angle XDE = 30°$. Find $m\angle EXT$.



Answer: 165°

C. Given $\angle A$ is a right angle, $DC = GC$ and $TO = TG$. Find $m\angle DGO$.



Answer: 45°

Meet 1

EVENT 3: Algebra I – Numbers

Include: (1) Evaluation
(2) Order of operations
(3) Patterns (recursive sequences must be stated)
(4) Number systems
(5) Bases other than 10

Exclude: (1) Inequalities
(2) Radical substitutions
(3) Non-integer exponents

Sample Problems:

A-1. What is the fractional form of the decimal number $0.266666...$?
Answer: 4/15

A-2. If $x\#y$ is defined to be $\frac{x+y}{x-y}$, what is the value of $(5\#3)\#2$?
Answer: 3

A-3. If $x=5$, $y=2$ and $z=3$, evaluate $\left(\frac{6y}{z}\right)^3+3x^2$.
Answer: 139

B-1. The first two terms of a sequence are 3 and 10. Starting with the 3rd term, each term of the sequence is obtained by doubling the previous term and subtracting the term before that. What is the 100th term of this sequence?
Answer: 696

B-2. The Egyptians used the following symbols: 𓏺 for 1, 𓎆 for 10, and 𓍢 for 100. What would the number 𓍢𓍢𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺 be if written in base 2?
Answer: 11110101

C-1. Convert the Roman numeral DCLXVII to base 16.
Answer: 29B

C-2. What is the 10th hexagonal number?
Answer: 190

C-3. If $r=3$, $s=-2$, and $t=1$, evaluate $\frac{-s^6+(r+1)^3}{(rt+2)^2}$.
Answer: 0

Meet 1

EVENT 4: Trigonometry - Identities

Include: (1) Reduction formulas
(2) Addition and subtraction laws, half-angle laws, doubling laws

Exclude: (1) Sums to products, products to sums
(2) Proofs

Sample Problems:

A. Determine the value of $\sin(-105°)$.

Answer: $-\dfrac{\sqrt{6}+\sqrt{2}}{4}$

B. Evaluate: $\dfrac{(\sin 45°)(\cos 15°)}{\cos 30°} - \dfrac{(\cos 45°)(\sin 15°)}{\sin 60°}$

Answer: $\dfrac{\sqrt{3}}{3}$

C-1. Find the value of $\tan(x-y)$, where $\cos x = -\dfrac{\sqrt{2}}{2}$, $\sin y = \dfrac{\sqrt{3}}{2}$, $\dfrac{\pi}{2} < x < \pi$, $0 < y < \dfrac{\pi}{2}$.

Answer: $2+\sqrt{3}$

C-2. Express as a single function of A:

$$\sec\left(\frac{\pi}{2}-A\right)+\tan(A-\pi)-\cos\left(A-\frac{3\pi}{2}\right)-\cot\left(\frac{\pi}{2}-A\right)-\cos\left(\frac{\pi}{2}-A\right)$$

Answer: $\csc A$ or $\dfrac{1}{\sin A}$

Meet 1

EVENT 5: Algebra I - Sets

Notation: Complement of A: $\overline{A}$, A', or A^C
Cardinality (number of elements) of A: $n(A) =$ ___. (limited to finite sets)
Not acceptable: $\{\varnothing\}$, $\{\mathbb{R}\}$

Include: (1) Set operations
(2) Complements
(3) Venn Diagrams
(4) Cardinality

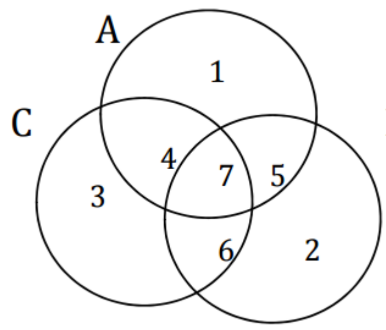
Exclude: (1) Inequalities
(2) Equations
(3) Answers which involve shading of diagrams

Sample Problems:

A. List the members of the set $\{s : s \text{ is a subset of } \{0, 1\}\}$.

Answer: $\{0, 1\}$, $\{0\}$, $\{1\}$, $\varnothing$

B. $U = A \cup B \cup C$ and regions 1, 2, 3, 4, 5, 6, and 7 are disjoint subsets of U. Which of these seven regions are contained in the set $A^C \cap (B \cup C)^C$?



Answer: none

C. At registration for fall semester, 100 students signed up for English, 80 for math, 60 for science. 40 signed up for math and English but not science. 10 signed up for science and English but not math. 35 signed up for math and science but not English. If a special class is to be formed made of all those students taking English, with no math or science, what are the most and least numbers of students that the class may have?
Answer: Most = 50; Least = 45

Meet 1

EVENT 6: Algebra II – Fractional Expressions & Equations

Notes: (1) If more than one letter is used in the problem, indicate which letter is the variable to be solved and which letters are constants.
(2) Assume that denominators are non-zero

Include: (1) Work word problems
(2) Sum and difference of cubes
(3) Complex fractions

Sample Problems:

A-1. Solve: $\dfrac{8x-7}{x+14}=\dfrac{x+14}{8x-7}$

Answer: 3, $-\dfrac{7}{9}$

A-2. Simplify: $\dfrac{5c^2-5c}{4a^3}\cdot\dfrac{c^2-9c-10}{4c-40}\div\dfrac{2-2c^2}{a}$

Answer: $-\dfrac{5c}{32a^2}$

B-1. Solve for y, where m is a constant: $\dfrac{m-2y}{m+y}+\dfrac{m+4y}{2m-y}=2$

Answer: $-\dfrac{m}{4}$, $\dfrac{m}{2}$

B-2. Simplify: $\dfrac{1}{a^2-ba-ca+bc}+\dfrac{1}{b^2-bc-ab+ac}+\dfrac{1}{c^2-ac-bc+ab}$

Answer: 0

B-3. Tom and Huck paint a fence for four hours, after which Jim helps them and they finish two hours later. If Jim had not helped them, it would have taken them five more hours to paint the fence. How long would it take Jim to pain the fence alone?
Answer: 6 hours

C-1. The sum of two numbers is 78. If the larger number is divided by the smaller number, the quotient is 5 and the remainder is 6. Find the absolute value of the difference of the two numbers.
Answer: 54

C-2. Simplify: $\dfrac{\dfrac{1}{p(p-1)}+\dfrac{1}{p(p+1)}}{\dfrac{1}{p^2-1}-\dfrac{1}{p^2+1}}$

Answer: p^2+1

Meet 2

EVENT 1: Algebra II – Ratio, Proportion & Variation

Sample Problems:

A. Kelly earns \$13.50 for working six hours. If the amount of money she earns is directly proportional to the number of hours she works, how much does she earn for working ten hours?
Answer: \$22.50

B. If y varies directly as t, and $y = m$ when $t = n$, find y when $t = p$.
Answer: $y = \frac{mp}{n}$

C. P varies directly as T and inversely as V. If T is increased by 25% and V is decreased by 20%, by what percent will P increase?
Answer: 56.25% or $56\frac{1}{4}\%$

Meet 2

EVENT 2: Geometry – Similar Triangles

Include: Explicit instructions for unit of measure of the answer. Suggestion: write the unit of measure on the answer blank if only the numerical value is to be graded.

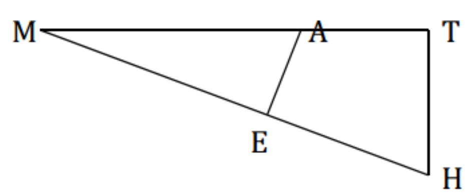
Exclude: (1) Area
(2) Circles

Sample Problems:

A. Given: $\triangle ABC$; D is on $\overline{AB}$ and E is on $\overline{AC}$ with $\overline{DE} \parallel \overline{BC}$; $AB = 5$ inches; $AC = 7$ inches; $EC = 3$ inches. Find AD.

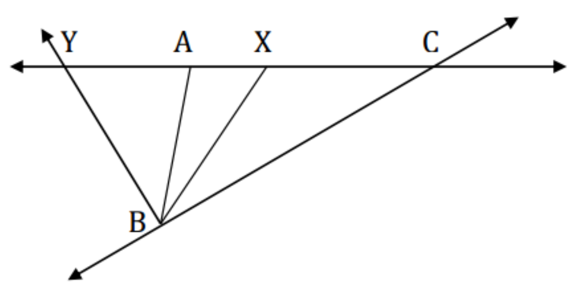
Answer: $2\frac{6}{7}$ or $\frac{20}{7}$ inches

B. Given: $\triangle MTH$ with $\overline{MT} \perp \overline{TH}$; $\overline{AE} \perp \overline{MH}$; $MA = 5$; $AT = 3$; $TH = 6$. Find $ME \cdot MH$.



Answer: 40

C. In this figure, $AB = 6$, $BC = 12$, $AC = 16$, $\overrightarrow{BX}$ bisects $\angle ABC$ and $\overrightarrow{BY}$ bisects an exterior angle of $\triangle ABC$. Find YC.



Answer: 32

Meet 2

EVENT 3: Algebra I – Linear Equations with One Variable

Include: Equations involving one variable and several constants

Exclude: Word problems

Sample Problems:

A-1. Solve for x: $\frac{3x}{4} - \frac{2x}{3} = \frac{x}{2} - 1$

Answer: $\frac{12}{5}$

A-2. Solve for x (a, b, c and d are constants): $\frac{3abcx}{2d} = 5$

Answer: $\frac{10d}{3abc}$

B. Solve for x. Write your answer as a reduced fraction.

$$0.06(0.003x + 0.005) - 0.04 = 0.07(0.002)$$

Answer: $\frac{664}{3}$

C. Solve for x: $-\frac{3}{4}\left(x + \frac{1}{3}\right) + \frac{2}{3}\left(x - \frac{1}{2}\right) + \frac{3}{5}\left(x - \frac{1}{3}\right) = \frac{7}{60}$

Answer: $\frac{54}{31}$

Meet 2

EVENT 4: Trigonometry – Equations & Graphs

Include: (1) Variable will have a restricted domain that will indicate whether the answer should be in degrees or radians.
(2) May include trigonometric graphs

Exclude: (1) Inverses: problems should be solvable without the use of the inverse functions (NO Arcsin, Arccos, etc.)
(2) Sums to products, products to sums

Sample Problems:

A-1. Solve for x such that $0 \le x < 2\pi$: $2\cos^2 x - \cos x = 1$
Answer: 0, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$

A-2. Solve for x such that $0° \le x < 360°$: $\sin 2x = 1$
Answer: $45°$, $225°$

B. Solve for x such that $0° \le x < 360°$: $\sin x - \sqrt{3}\cos x = 1$
Answer: $90°$, $210°$

C. Solve for x such that $0° \le x < 360°$: $2\cos^2 \frac{x}{2} = \sin^2 x$
Answer: $90°$, $180°$, $270°$

Meet 2

EVENT 5: Geometry – Logic

Notation: Conditional: $\rightarrow$ Biconditional: $\leftrightarrow$
And: $\wedge$ Or: $\vee$
Negation: ~
Lower case letters to represent statements

Sample Problems:

A. Which of the following is (are) true?

a) $q \rightarrow p$ is the converse of $p \rightarrow q$.
b) $q \rightarrow p$ is the inverse of $p \rightarrow q$.
c) $\sim p \rightarrow \sim q$ is the contrapositive of $q \rightarrow p$.
d) $\sim q \rightarrow \sim p$ is the contrapositive of $q \rightarrow p$.
e) $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$.
f) $\sim p \rightarrow \sim q$ is the converse of $\sim q \rightarrow \sim p$.

Answer: a, c, e, f

B. Start with the theorem, "If two angles of a triangle are congruent, the triangle is isosceles," and examine the following statements:

a) If two angles of a triangle are not congruent, the triangle is not isosceles.
b) The base angles of an isosceles triangle are congruent.
c) If a triangle is not isosceles, then two of its angles are not congruent.
d) A necessary condition that two angles of a triangle be congruent is that the triangle is isosceles.

Which of these four statements is (are) logically equivalent to the theorem?
Answer: c, d

C-1. Emerson, Lake and Palmer are different heights. Arrange them from the tallest to the shortest if <u>only one</u> of the following statements is true.

a) Emerson is the tallest.
b) Lake is not the tallest.
c) Palmer is not the shortest.

Answer: tallest: Lake // middle: Palmer // shortest: Emerson

C-2. What conclusion can be drawn using <u>all</u> of the following statements? Express your answer <u>without</u> grouping symbols.

a) $p \vee q$
b) $\sim p$
c) $q \rightarrow (a \vee \sim b)$
d) $\sim a \wedge e$
e) b is a necessary condition for $\sim c \wedge d$.

Answer: $c \vee \sim d$

Meet 2

EVENT 6: Algebra II – Absolute Values & Inequalities

Exclude: Problems leading to inequalities/equations involving more than two distinct absolute value expressions.

Sample Problems:

A. Solve for all values x that satisfy $|x| \le \frac{1}{x}$

Answer: $0 < x \le 1$

B. Solve for x: $x^3 - x^2 - 2x < 0$

Answer: $x < -1$ or $0 < x < 2$

C. Solve for x: $|3-2x| - |6+3x| > |2+x|$

Answer: $-\frac{11}{2} < x < -\frac{5}{6}$

Meet 3

EVENT 1: Algebra II – Exponents & Radicals

Note: $\sqrt{x^2} = |x|$

Special Directions:

(1) Leave all answers in simplest radical form.
(2) Assume all radical expressions are defined over the set of real numbers.

Include: (1) Radical exponents
(2) Radicals with index equal to 2, 3, 4, …
(3) Simple problems which involve rationalizing the denominator
Example: $\dfrac{1}{\sqrt[3]{x} - \sqrt[3]{y}}$

Exclude: Problems involving simplifications of the type: $\dfrac{1}{\sqrt[5]{x} - 1}$

Sample Problems:

A-1. Simplify: $\dfrac{8^{-2} + 1^0 - 1^4}{2^{-8}}$

Answer: 4

A-2. Simplify: $\sqrt{x^2 - 2x + 1}$

Answer: $|x - 1|$

B. Solve for x: $8^{\frac{1}{6}} + x^{\frac{1}{3}} = \dfrac{7}{3 - \sqrt{2}}$

Answer: 27

C. Simplify the following expression: $\dfrac{1}{x + \sqrt{x}} + \dfrac{1}{\sqrt{x} - x} + \dfrac{1}{\sqrt{x}}$

Answer: $\dfrac{3\sqrt{x} - x\sqrt{x}}{x - x^2}$

Meet 3

EVENT 2: Geometry – Right Triangles

Include: (1) Pythagorean Theorem
(2) Special right triangles ($30°$-$60°$-$90°$ and $45°$-$45°$-$90°$)

Exclude: (1) Area
(2) Quadrilateral relationships
(3) Circles

Sample Problems:

A. The hypotenuse of a right triangle is $4\sqrt{10}$, and the ratio of the legs is $1:3$. Find the perimeter of the triangle.
Answer: $16+4\sqrt{10}$

B. In $\triangle ABC$, $\angle C$ is obtuse, $m\angle A = 30°$, $AC = 4$ and $AB = 3\sqrt{3}$. Find BC.
Answer: $\sqrt{7}$

C. In $\triangle ABC$, $AB = BC$ and $\angle B = 90°$. $\overrightarrow{AD}$ bisects $\angle A$ and intersects $\overline{BC}$ at D. If $BD = 4$ inches, find the length of the longest side of the triangle.
Answer: $8+4\sqrt{2}$

Meet 3

EVENT 3: Algebra I – Age, Motion & Number Problems

Include: (1) Basic motion problems utilizing the concept that $d = rt$.
(2) Problems considering rate of current or a wind factor.
(3) Problems utilizing a one-variable linear equation to solve.
(4) Problems utilizing a one-variable quadratic equation to be solved by factoring.
(5) Explicit instructions for unit of measure of the answer. (i.e. miles per hour versus miles per minute, seconds versus minutes, hours versus a time of day, miles verses feet, etc.)

Exclude: Problems utilizing a two-variable system of linear equations to solve.

Sample Problems:

A. Kimo is four years older than Dan. In two years, their ages will total 50. How old is Dan?
Answer: 21

B. Find three consecutive odd integers such that the sum of the greatest and twice the least is 25.
Answer: 7, 9, 11

C. A speedboat has a speed of 27 miles per hour in still water. If it requires $1\frac{1}{8}$ hours for the boat to go 15 miles downstream and then return, what is the rate of the current in miles per hour?
Answer: 3 mph

Meet 3

EVENT 4: Trigonometry – Inverse Functions including Triangles

Notation: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, $\sec^{-1}$, $\csc^{-1}$, $\cot^{-1}$, arcsin, arccos, arctan, arcsec, arccsc, arccot

Sample Problems:

A-1. In $\triangle ABC$, if $a = 4$, $b = 5$, and $C = \arctan\left(\frac{24}{7}\right)$, find the area of the triangle.

Answer: $\frac{48}{5}$

A-2. Evaluate and leave in simplest radical form: $\cos\left(\text{arc Tan}\frac{3}{4} - \text{arc Sin}\frac{1}{2}\right)$

Answer: $\frac{4\sqrt{3}+3}{10}$

A-3. Evaluate $\text{Cos}^{-1} 2$.

Answer: Undefined or Does not exist

B-1. In $\triangle ABC$, if $a = 4$, $b = 5$ and $c = 6$, compute $\tan C$.

Answer: $3\sqrt{7}$

B-2. Solve for x if $\text{Arcsin}\, x = \text{Arctan}\sqrt{1-x^2}$.

Answer: $\frac{-1+\sqrt{5}}{2}$

C-1. In $\triangle ABC$, if $A = \arctan\left(\frac{4}{3}\right)$, $B = \arctan\left(\frac{12}{5}\right)$, and $a = 26$, find c.

Answer: 28

C-2. Solve for θ in radians if $\theta = \text{Tan}^{-1}\left(\frac{1}{2}\right) + \text{Tan}^{-1}\left(\frac{1}{5}\right) + \text{Tan}^{-1}\left(\frac{1}{8}\right)$.

Answer: $\frac{\pi}{4}$

Meet 3

EVENT 5: Algebra I – Operations with Polynomials

Include: (1) Addition, subtraction, multiplication, and division of polynomials where answers contain a reasonable number of terms.
(2) Explicit directions for answers.

Exclude: (1) Negative exponents
(2) Equations and word problems

Sample Problems:

A. Express as a polynomial in simplest form in x: $(x-y)(x+y)^2$
Answer: $x^3+x^2y-xy^2-y^3$

B. Expand and simplify by combining like terms: $m^c(m^d+p^c)+p^c(m^c+p^d)$
Answer: $m^{c+d}+2m^cp^c+p^{c+d}$

C. Divide the following and express the result in simplest form and in the form $\text{quotient}+\frac{\text{remainder}}{\text{divisor}}$: $\frac{2x^2+x^3+12-5x}{x+4}$
Answer: x^2-2x+3

Meet 3

EVENT 6: Algebra II – Relations & Functions

Notes: (1) Any set of ordered pairs of numbers is called a relation. The set of all first coordinates is the domain; the set of all second coordinates is the range.

(2) Notation to be used for function operations:

(a) Composition: $[f \circ g](x) = f[g(x)]$

(b) Addition/Subtraction: $[f \pm g](x) = f(x) \pm g(x)$

(c) Multiplication $[f \cdot g](x) = f(x) \cdot g(x)$

(d) Division $\left[\frac{f}{g}\right](x) = \frac{f(x)}{g(x)}, \ g(x) \neq 0$

(3) Domains

(a) for $f \pm g$, $f \cdot g$: $\{x : x \in D_f \cap D_g\}$

(b) for f/g: $\{x : x \in D_f \cap D_g \text{ and } g(x) \neq 0\}$

(c) for $f \circ g$: $\{x : x \in D_g \text{ and } g(x) \in D_f\}$

(4) If a problem involves an inverse, specify if it is an inverse function or an inverse relation.

(a) Inverse relation: The inverse relation is the relation obtained by interchanging the coordinates in each ordered pair in the given relation

(b) Inverse function: If a given relation and its inverse relation are both functions, they are called inverse functions.

Exclude: Trigonometric functions

Sample Problems:

A. If $f(x) = x^3 - 2x$, find $f(3t)$.

Answer: $27t^3 - 6t$

B. If $f(x) = 2x$ and $g(x) = \frac{1}{2}x$ and $[f \circ g](x) = 6$, find the value of x.

Answer: 6

C. Find the inverse relation for $\{(x, y) : y = 2x^2 - 3\}$. Indicate any restrictions on the domain of the inverse relation. Express your answer in the form $\{(x, y) : y = \cdots\}$

Answer: $\left\{(x, y) : y = \pm\frac{1}{2}\sqrt{2x+6}, \ x \geq -3\right\}$

Meet 4

EVENT 1: Algebra II – Exponents & Logarithms

Note: $\log x = \log_{10} x$

Exclude: Problems requiring tables

Sample Problems:

A. Solve for x: $3^{\frac{x}{3}} = \frac{1}{27^2}$

Answer: -18

B. Solve for x: $3\log_7 x + \log_7 1 - \log_{13} 13 = 2$

Answer: 7

C. Solve for x: $x^{\log x} = 1000x^2$

Answer: 1000, $\frac{1}{10}$

Meet 4

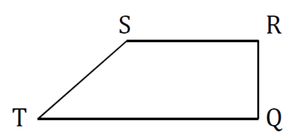
EVENT 2: Geometry – Convex Polygons

Include: (1) Special quadrilaterals such as parallelograms, rectangles, rhombi, squares, trapezoids and kites
(2) Problems involving areas
(3) Similarity, ratios and proportions
(4) Regular polygons, apothem
(5) Explicit instructions for unit of measure of the answer
Suggestion: Write the unit of measure on the answer blank if only the numerical value is to be graded.

Exclude: Circles

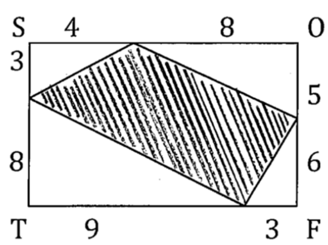
Special Instructions: Find exact answers (i.e. simplest radical form)

A-1. Given trapezoid $RSTQ$ with bases $\overline{QT}$ and $\overline{RS}$, $RQ = RS = k$, $m\angle R = 60°$, and $m\angle T = 45°$. Find QT in terms of k.



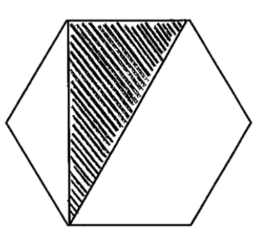
Answer: $\frac{1+\sqrt{3}}{2}k$

A-2. Given that $SOFT$ is a parallelogram with $\angle S$ is a right angle and the lengths as given in the diagram, find the area of the shaded region.



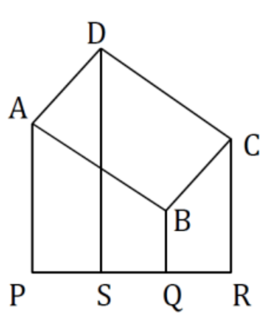
Answer: 61

B-1. Given a regular hexagon with sides r units long, find the area of the shaded region.



Answer: $8\sqrt{3}$

B-2. $ABCD$ is a parallelogram and $\overline{AP}$, $\overline{BQ}$, $\overline{CR}$, and $\overline{DS}$ are perpendicular to $\overline{PR}$. $AP = 12$ inches, $DS = 16$ inches, $CR = 10$ inches, $PS = 5$ inches, and $SQ = 2$ inches. Find the area of $ABCD$.

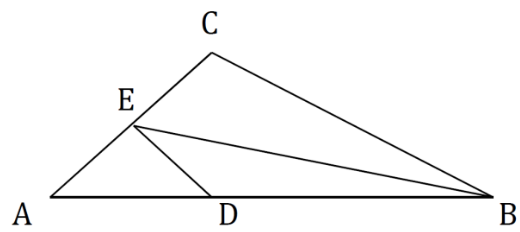


Answer: 58 square inches

C-1. Given parallelogram $STAR$ with diagonal $RT = 25$, $RA = 14$, and $TA = 16$. The angle bisector $\overrightarrow{SE}$ of $\angle S$ intersects $\overline{RT}$ at E. Find RE.

Answer: $\frac{40}{3}$

C-2. Given $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$, $DB = 2DA$, and the area of $\triangle DBE = 9$ cm². Find the area of $\triangle ABC$.



Answer: 40.5 square units

Meet 4

EVENT 3: Algebra I – Factoring Expressions and Equations

Include: (1) Factoring trinomials
(2) Factoring by grouping and extracting common factors

Exclude: (1) Factoring of sums or differences of cubes
(2) Word problems

Sample Problems:

A. Solve for m: $5m^3 - 2m = 3m^2$
Answer: 0, 1, $-\frac{2}{5}$

B. Factor completely: $4 - a^2 - 2ab - b^2$
Answer: $(2 + a + b)(2 - a - b)$

C. Solve for x: $x^3 - 3x^2 - cx^2 + 3cx - 2c^2x + 6c^2 = 0$
Answer: 3, $2c$, $-c$

Meet 4

EVENT 4: Trigonometry – Polar Coordinates & DeMoivre's Theorem

Note: Refer to "Guidelines for Forms of Answers" Part A for simplification.
Acceptable: $2+3i$, 4, 0, $-2i$
Not acceptable: $0+0i$, $4+0i$, $0-2i$

Include: (1) Relationships between polar and rectangular coordinates
(2) Specific directions on the form of equations
(3) Products, quotients and rational powers in $r(\cos\theta + i\sin\theta)$ or $r\operatorname{cis}\theta$ form

Exclude: Systems of polar equations

Sample Problems:

A. Change from polar coordinates to Cartesian coordinates: $\left(-3, \dfrac{2\pi}{3}\right)$.

Answer: $\left(\dfrac{3}{2}, -\dfrac{3\sqrt{3}}{2}\right)$

B. Transform the polar equation $r = \dfrac{1}{1-\cos\theta}$ into an algebraic equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

Answer: $y^2 - 2x - 1 = 0$

C. Find all of the cube roots of $2i$. Express your answer in $r\operatorname{cis}\theta$ form, where $0 \le \theta < 2\pi$.

Answer: $\sqrt[3]{2}\operatorname{cis}\dfrac{\pi}{6}$, $\sqrt[3]{2}\operatorname{cis}\dfrac{5\pi}{6}$, $\sqrt[3]{2}\operatorname{cis}\dfrac{3\pi}{2}$

Meet 4

EVENT 5: Algebra I – Linear Inequalities

Include: (1) Linear inequalities as long as the inequality is linear in the variable being solved for (i.e. "Solve for x: $b - a^2 x > c$")

(2) Explicit instructions for the form of the desired answer:

(a) Solve for x

(b) State answer in the form $\{x : \ldots\}$

(c) State answer in the form $\{x | \ldots\}$ utilizing "and" or "or" as necessary to state answer as one solution set

(3) Word problems

Exclude: (1) Absolute value problems

(2) Variables in the denominator

Sample Problems:

A. Solve for x: $\frac{4x+3}{15} - \frac{2x-3}{9} > \frac{3x+2}{3} - x$

Answer: $x > 3$

B. Find the solution set for $2r - 1 \le 2r + 8 \le 2r + 4$

Answer: $\varnothing$ or $\{\ \}$

C. Find the solution set. State your answer in the form $\{c : \ldots\}$

$$\{c : 4c + 4 \ge c + 10\} \cup \{c : 3c - 3 \ge 2c - 9\}$$

Answer: $\{c : c \ge -6\}$

Meet 4

EVENT 6: Calculus – Limits and Derivatives

Include: (1) Computing limits
(2) Continuity & differentiability
(3) Average rates of change
(4) Computing derivatives
(5) Implicit differentiation
(6) Applications of derivatives: optimization, related rates, linearization, particle motion
(7) Curve analysis: increasing, decreasing, extrema, concavity, inflection points
(8) The Mean Value Theorem
(9) Average rate of change

Exclude: (1) Problems where the answer is ∞ or $-\infty$
(2) Indeterminate forms that are not $0/0$ or ∞/∞
(3) Parametric & polar functions

Note: (1) When a limit does not exist, "does not exist," or "non-existent," are acceptable answers, "DNE" is not acceptable.

Sample Problems:

A-1. Compute: $\lim\limits_{x\to 2} \dfrac{x^3-8}{x^2-4}$
Answer: 3

A-2. Compute: $\lim\limits_{x\to 0} \dfrac{x+\sin x}{x}$
Answer: 2

A-3. Compute: $\lim\limits_{x\to\infty} \sin^{-1}\left(\dfrac{x+3}{2x+5}\right)$
Answer: $\pi/6$

A-4. Compute: $\lim\limits_{x\to\infty} \dfrac{x^2}{2^x}$
Answer: 0

A-5. Compute: $\lim\limits_{h\to 0} \dfrac{\tan\left(\frac{\pi}{4}+h\right)-\tan\frac{\pi}{4}}{h}$
Answer: 2

A-6. Find the average rate of change of $f(x)=\sqrt{x}$ from $x=1$ to $x=9$.
Answer: $\dfrac{1}{4}$

B-1. Compute: $\dfrac{d}{dx}(-2x\cos 2x+\sin 2x)$
Answer: $4x\sin 2x$

B-2. Given the function $f(x)=\sqrt{x^2+9}$, use the linearization of $f(x)$ at $x=4$ to estimate $f(4.2)$.
Answer: 5.16

B-3. If $f(x)=xe^{-x}$, on what interval(s) is f concave up?
Answer: $x>2$

B-4. Given the curve $x^2+xy+y^3=7$, find the slope of the tangent line at the point $(2, 1)$.
Answer: -1

B-5. Let $f(x)=\begin{cases} ax^2+bx, & x<1 \\ x^3, & x\geq 1 \end{cases}$

Find the values of a and b so that f will be continuous and differentiable at $x=1$.

Answer: $a=2$, $b=-1$

B-6. Find the value(s) of c that satisfy the conclusion of the Mean Value Theorem for $f(x)=\sqrt{x}$ on the interval $[1, 9]$.

Answer: 4

C-1. A right circular cone is formed by revolving a right triangle with hypotenuse 2 cm about one of its legs. If x represents the height of the cone, find the value of x (in cm) that will maximize the volume of the cone.

Answer: $\dfrac{2\sqrt{3}}{3}$

C-2. If $y=\dfrac{x}{x^2+1}$, for what values of x is $\dfrac{d^2y}{dx^2}=0$?

Answer: 0, $\pm\sqrt{3}$

C-3. A particle's position on the x-axis is given by $x(t)=t^3-6t^2+9t-4$ for $t\geq 0$. For what values of t is the particle speeding up?

Answer: $1<t<2$ or $t>3$

C-4. The length of a rectangle is increasing at a rate of 3 inches per minute and the width is decreasing at a rate of 2 inches per minute. At the moment when the length is 8 inches and the width is 6 inches, how fast is the angle formed by the length and the diagonal changing? Give answer in radians per minute.

Answer: $-\dfrac{17}{50}$

Meet 5

EVENT 1: Algebra II – Complex Numbers

Notes: (1) All answers should be expressed in simplest rectangular form
(2) Refer to "Guidelines for Forms of Answers" Part A for simplification
Acceptable: $2+3i$, 0, $6i$
Not acceptable: $4+0i$, $0-2i$

Include: (1) All operations of complex numbers: addition, subtraction, multiplication, division, finding square roots, reciprocals, and absolute values
(2) Equations of degree ≥ 2 with real coefficients

Exclude: (1) Trigonometry
(2) Vectors

Sample Problems:

A. If $a=6+3i$ and $b=-3-2i$, find z if $b+ai=zi$.
Answer: $4+6i$

B. Find the reciprocal of z if $z=\frac{2}{5}+\frac{\sqrt{3}}{3}i$.
Answer: $\frac{30}{37}-\frac{25\sqrt{3}}{37}i$

C. Find z: $(3-i)z+2-2i=-6i$
Answer: $-\frac{1}{5}-\frac{7}{5}i$

Meet 5

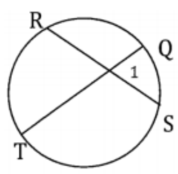
EVENT 2: Geometry – Circles with Arcs, Chords, Secants, & Tangents

Include: (1) Central, inscribed, tangent, and secant angles
(2) Power of the point theorems (tangent, secant, chord segments)

Exclude: (1) Areas of any type
(2) Arc length

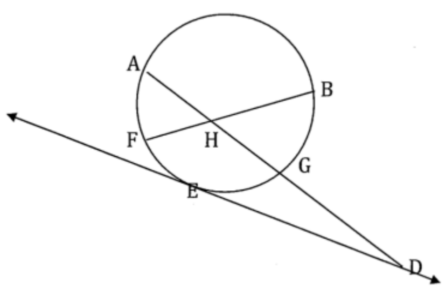
Special Instructions: Find exact answers (i.e. simplest radical form)

A. Given two intersecting chords $\overline{RS}$ and $\overline{TQ}$, if $m\angle 1 = x^2$, the measure of arc $RT = 3x$, and $m\widehat{QS} = x + 30$, find x.



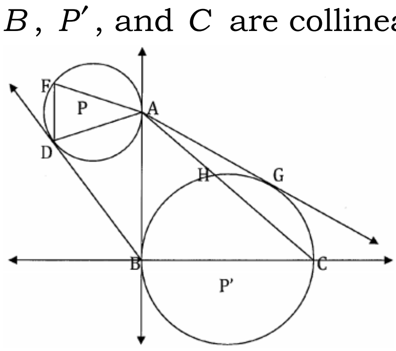
Answer: 5

B. Given that chord $\overline{FB}$ bisects $\overline{AG}$ at H and $\overline{ED}$ is tangent to the circle at E. If $FH = 3$, $BH = 12$, and $GD = 3$, find DE.



Answer: $3\sqrt{5}$

C. Given circles P and P' with common tangent $\overline{AB}$, $\overline{AG}$ is tangent to circle P' at G; $\overline{BD}$ is tangent to circle P at D; $m\widehat{GC} = 40°$, $m\widehat{HG} = 30°$; B, P', and C are collinear and $m\angle BAG = m\angle DBA$, find $m\widehat{DFA}$.



Answer: 220°

Meet 5

EVENT 3: Algebra I – Systems of Linear Equations

Include: (1) Systems involving word problems
(2) Systems with at most two equations, two unknowns
(3) Explicit directions for the form of the answer, i.e.:

(a)	find the solution set	Answer:	$\{(3, 4)\}$
(b)	find the point of intersection	Answer:	$(3, 4)$
(c)	solve for x and y	Answer:	$x=3$, $y=4$

Exclude: (1) Word problems involving mixtures or percents
(2) Systems of the form: $\frac{1}{x}+\frac{1}{y}=7$; $\frac{2}{x}+\frac{3}{y}=16$

Sample Problems:

A. Solve for x and y:

$$\frac{3x+3y}{3}-\frac{3x+2y}{5}=-\frac{1}{5}$$
$$\frac{x+y}{3}-\frac{x+2y}{3}=1$$

Answer: $x=4$, $y=-3$

B. Find the point(s) of intersection: $6x-15y=12$; $4x-8=10y$
Answer: all points on $2x-5y=4$ or $\{(x,\ y): 2x-5y=4\}$

C. The width of a rectangle is two-thirds the length. If 4 inches were added to the width and 4 inches were subtracted from the length, the rectangle would be a square. Find the dimensions of the rectangle.
Answer: length: 24 inches // width: 16 inches

Meet 5

EVENT 4: Analytic Geometry – Lines & Circles

Exclude: (1) Vectors
(2) Degenerate cases
(3) Use of the term "normal"

Note: Be sure to state your final answer in the form described in each problem

Sample Problems:

A. If the angle of inclination of a line in the xy-plane is $120°$ and $P(-1, 0)$ is a point on the line, find the equation of the line. Write your answer in the form $y = Ax + B$, where A and B are real numbers.
Answer: $y = -\sqrt{3}x - \sqrt{3}$

B. Find the equation of the line $Ax + By + C = 0$, where A, B and C are integers, that is tangent at $(-3, 1)$ to the circle whose equation is $x^2 + y^2 + 8x - 2y + 16 = 0$.
Answer: $x + 3 = 0$

C. Given two lines parallel to the line $3x - 4y = 6$ such that the undirected length of the perpendicular segment drawn from the origin to them is 3. Find the equations. Write your answers in the form $Ax + By + C = 0$, where A, B and C are real numbers and A and B are not both zero.
Answer: $3x - 4y + 15 = 0$ and $3x - 4y - 15 = 0$

Meet 5

EVENT 5: Algebra I – Statistics

Include: (1) Measures of central tendency: mean, median, mode
(2) Weighted averages
(3) Measures of spread: range, interquartile range, standard deviation
(4) Percentiles (don't include top number when calculating percentiles)
(5) Empirical rule (68%-95%-99.7%)
(6) Lines of best fit including error
(7) Standard deviation and variance (note: indicate sample or population)
(8) Stem-and-leaf, box-and-whiskers, histograms, dot plots, pie charts

Exclude: (1) Problems requiring permutations and combinations
(2) Nonlinear regression/models
(3) Ogives
(4) Residuals
(5) Probability

Sample Problems:

A-1. Mr. Simmons administered a fitness exam at the beginning and end of the semester. Each student did as many pull-ups as he/she could do in one minute. The results of the pre- and post-exam are shown in the stem-and-leaf plot below. Note that not all students were present on both days of the fitness exams. Find the difference between the median scores of the pre- and post-exams.

Pre-Exam		Post-Exam
4, 0, 0	2	
9, 8, 7, 7, 7, 6	2	
4, 4, 4, 4, 4, 3, 3, 1, 0	3	4
8, 8, 5, 5	3	5, 9
2, 0	4	4
6	4	5, 5, 8, 8
	5	1, 1, 1, 2, 2, 2, 4
	5	6, 7, 8, 9, 9, 9
	6	0, 0, 2, 3, 4
	6	6, 6, 7

Answer: 21

A-2. Find the interquartile range for the data set:
$\{2, 3, 6, 4, 1, 9, 3, 8, 6, 2, 2, 10\}$

Answer: 5

A-3. On January 1, Jon decided to start running for one hour every day. On some days, he uses a smart phone to record the number of miles that he runs. The data below shows Jon's data. If Jon uses the data he collected on January 1 and January 11 to create a line of best fit to model his data, compute the error for the data collected on January 31.

Number of days since January 1	0	10	30
Number of miles ran in one hour	1.8	2.3	6.4

Answer: 3.1 miles

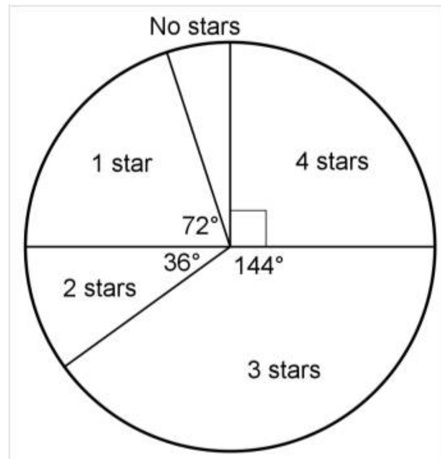
A-4. Find the sample standard deviation for the data set $\{5, 6, 10\}$.

Answer: $\sqrt{7}$

B-1. The data set $\{3, 4, 9, 12, 8, 16, 7, 15, x\}$, where x is a whole number, has median 9 and range 13. Find all possible values for the upper quartile of the data.

Answer: 13.5, 14, 14.5, 15, 15.5

B-2. 20 people were randomly selected to be a part of a screening room in which they got to view an upcoming Jennifer Lawrence movie. Each person was then asked to give their overall rating, either 0, 1, 2, 3 or 4 out of 4 stars. The pie chart shows the ratings given by audience members of the screening room. Find the positive difference between the median rating and mean rating.



Answer: 0.4

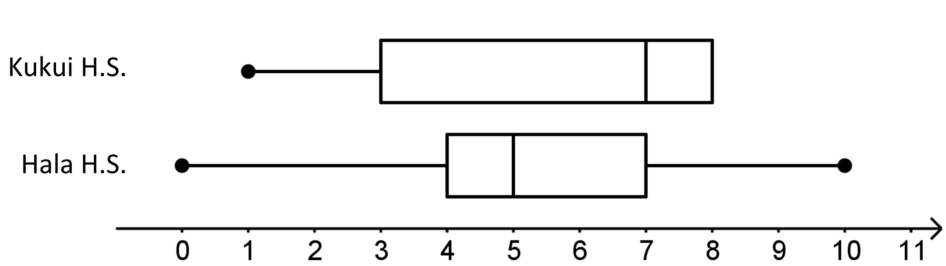
C-1. The distribution of weights of chocolate chip cookies produced by a bakery is approximately normal. Out of the 2000 chocolate chip cookies that it pumps out every day, 320 cookies weigh less than 10.4 grams, and 1950 weigh more than 8.8 grams. What is the mean weight for the chocolate chip cookies produced by the bakery?

Answer: 12 grams

C-2. Mr. Mean administered his AP Statistics final exam yesterday. 8 boys and 16 girls were present. The only student who was absent and must make up the exam was Lance (a boy). After grading yesterday's stack of exams, Mr. Mean determines that the average score for the boys so far is 72% while the average score for the girls is 84%. Suppose Lance makes up the exam and with his score the class average is 80%, what is the new average score for the boys including Lance's score?

Answer: $72\frac{8}{9}\%$

C-3. In a study, 200 students from Hala High School and 300 students from Kukui High School were asked, "How many hours of sleep do you receive on average each night?" Students were only allowed to give whole number responses. The results of the poll are represented by the side-by-side box-and-whisker plot shown below. Find the minimum and maximum overall percentage of students from both schools who received at least 7 hours of sleep on average each night.



Answer: minimum 40%, maximum 64.6%

Meet 5

EVENT 6: Algebra II – Matrices & Determinants

Include: (1) Determinants

(a) 3×3 at most

(b) Notation to be used: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

(2) Matrices

(a) Elementary operations (i.e. $+$, $-$, $\times$, scalar multiplication)

(b) Inverses of 2×2 matrices

(c) Notation to be used: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(d) Transposes

Note: Use capital letters to name matrices and lower case letters to name scalars.

Sample Problems:

A. Find the inverse, A^{-1}, of the matrix $A=\begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix}$

Answer: $\frac{1}{4}\begin{bmatrix} 5 & -6 \\ -1 & 2 \end{bmatrix}$ or $\begin{bmatrix} \frac{5}{4} & -\frac{3}{2} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$

B. Determine x if $\begin{bmatrix} 1 & y \\ x & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} 1 & y \\ x & 2 \end{bmatrix}$

Answer: 2

C. Solve for x: $\begin{vmatrix} 1 & 2 & x \\ -2 & -1 & -2 \\ 3 & 1 & 4 \end{vmatrix}=-2$

Answer: -4

Meet 6

EVENT 1: Algebra II – Polynomial Equations

Notes: (1) All complex roots should be expressed in simplest rectangular form
(2) Refer to "Guidelines for Forms of Answers" Part A for simplification
Acceptable: $2+3i$, 0, $6i$
Not acceptable: $4+0i$, $0-2i$

Sample Problems:

A. Solve for x: $x^3+x^2+x+1=0$
Answer: 1, i, $-i$

B. Find all the roots of the equation $x^4+x^3-12x^2+32x-40=0$.
Answer: -5, 2, $1+i\sqrt{3}$, $1-i\sqrt{3}$

C. Find the lowest degree equation with integral coefficients having $-1+\sqrt{5}$ and -6 as two of its roots.
Answer: $x^3+8x^2+8x-24=0$

Meet 6

EVENT 2: Geometry – Plane Coordinate Geometry

Include: (1) Distance formula, midpoint formula, slope of lines, equations of lines
(2) Specific instructions for the form of the desired answer. Write out the form to be used, i.e. "Leave your answer in the form $y = mx + b$, where m and b are real numbers." Do not say: Leave your answer in standard form, etc.

Exclude: (1) Circles
(2) Areas

A. A parallelogram has vertices as follows: $(-4, -5)$, $(2, -5)$, $(4, 1)$, $(-2, 1)$. Find the length of the longer diagonal.
Answer: 10

B. For what value of k will the line $6x + ky = 12$ be perpendicular to the line $3x - 5y = 10$?
Answer: $\frac{18}{5}$

C. Find the equation of the locus of points equidistant from the points $(4, 1)$ and $(2, -5)$. Leave your answer in the form $Ax + By + C = 0$, where A, B and C are integers.
Answer: $x + 3y + 3 = 0$

Meet 6

EVENT 3: Algebra I – Fractions & Mixed Expressions

Include: (1) Variables
(2) Rational expressions

Exclude: (1) Equations
(2) Negative exponents, fractional exponents

Sample Problems:

A. Simplify: $\frac{3}{x+2}-\frac{5}{x-2}+\frac{2x-4}{4-x^2}$

Answer: $\frac{-4x-12}{x^2-4}$ or $\frac{4x+12}{4-x^2}$

B. Simplify: $\frac{3p-1}{3p}\div\frac{1+3p}{3p^2}\cdot\frac{3-2p-p^2}{p^2-p}$

Answer: $\frac{-3p^2-8p+3}{3p+1}$ or $-\frac{3p^2+8p-3}{3p+1}$

C. Simplify: $2+\cfrac{1}{1+\cfrac{2}{x+\cfrac{1}{x}}}$

Answer: $\frac{3x^2+4x+3}{x^2+2x+1}$

Meet 6

EVENT 4: Analytic Geometry – Parabolas, Ellipses & Hyperbolas

Special Instructions:
If the answer results in a nondegenerate conic section, answers should be in OML standard form or OML general form. Refer to OML Forms of answers.

Exclude: (1) Problems whose direct solutions depend on the derivative
(2) Rotations
(3) Degenerate conics

Sample Problems:

A. Find the equation of the directrix of the parabola with the equation $y^2 - 2y - 6x + 13 = 0$. Write your answer in the form $Ax + By + C = 0$, where A, B and C are integers.
Answer: $2x - 1 = 0$

B. A point moves so that the sum of its distances from (–2, 3) and (4, 3) is 10. Find the eccentricity of the curve traced by the point.
Answer: $\frac{3}{5}$

C. Find an equation of the hyperbola given:
a. the equations of its asymptotes are $y = \frac{1}{2}x - \frac{3}{2}$ and $y = -\frac{1}{2}x - \frac{5}{2}$
b. each focus is $2\sqrt{5}$ units from the center
c. the conjugate axis is parallel to the x-axis
Write your answer in OML general form ($Ax^2 + By^2 + Cx + Dy + E = 0$, where A, B, C, D and E are integers).
Answer: $x^2 - 4y^2 + 2x - 16y + 1 = 0$

Meet 6

EVENT 5: Geometry – Arc Lengths & Circular Regions

Include: (1) Inscribed and circumscribed polygons
(2) Sectors and segments of circles

Note: Find exact answers (i.e. simplest pi and/or radical form)

Sample Problems:

A. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are tangents to a circle at points B and C, respectively. Minor arc BC is 7π inches and the radius of the circle is 18 inches. What is the measure of $\angle BAC$ in degrees?
Answer: $110°$

B. An equilateral cross with perpendicular sides is inscribed in a circle. Each side of the cross is 2 cm. Find the area of the shaded region.

Answer: $10\pi - 20$ cm^2

C. From a point P, at a distance of m from the center O of a circle, two tangents are drawn to the circle meeting it at points A and B. A second circle is drawn with P as a center and $\overline{PA}$ as radius. Find the area bounded by $\overset{\frown}{AB}$ of the second circle and the radii $\overline{OA}$ and $\overline{OB}$ of the first circle if the measure of $\angle AOB$ is twice the measure of $\angle APB$.
Answer: $\dfrac{2\sqrt{3}-\pi}{8}$ m^2

Meet 6

EVENT 6: Algebra II – Arithmetic/Geometry Sequences & Series

Include: (1) Infinite geometric series where $|r| < 1$

(2) Sigma notation

Sample Problems:

A. Evaluate: $\sum_{k=1}^{10}(2k-1)$

Answer: 100

B. One-fourth of the air remaining in a cylinder is removed on each operation. How many cubic feet of air remains in the cylinder after the fifth operation if the cylinder contained 2048 cubic feet of air at the start?

Answer: 486 ft^3

C. Insert two positive numbers between 7 and 42 so that the first three numbers are in a geometric progression and the last three of the four numbers are in an arithmetic progression.

Answer: 14, 28

Meet 7

EVENT 1: Algebra II – Permutations, Combinations & Probability

Notation: (1) Permutations of n different elements taken r at a time: ${}_nP_r$, $P(n, r)$, or P_r^n

(2) Combinations of n different elements taken r at a time: ${}_nC_r$, $C(n, r)$, or $\binom{n}{r}$

Note: Problems should have reasonable answers

Include: (1) Basic properties and definitions (sum rule, product rule, complement rule, etc.)
(2) Probabilities involving permutations and combinations (not too heavily)
(3) Conditional probability
(4) Probability binomial theorem
(5) Independence, independent trials of processes
(6) Expected value
(7) Geometric probability

Exclude: (1) Statistics
(2) Random variables
(3) Continuous probabilities

Sample Problems

A-1. In the Kiwanis Club, there are 3 candidates for president, 4 for vice-president, 5 for secretary, but only 2 for treasurer. How many different slates are possible?
Answer: 120

A-2. What is the probability that when two cards are selected from a standard deck, at least one is a diamond?
Answer: $\frac{15}{34}$

A-3. Two 6-sided dice are rolled. What is the probability that the sum of the numbers showing is even?
Answer: $\frac{1}{2}$

B-1. An ice cream store has 31 flavors of ice cream and only sugar cones. How many different 3-scoop ice cream cones can you choose from if each scoop is a different flavor and the order of the flavors does not matter?
Answer: 4495

B-2. The Cardinals and the Ordinals are playing each other in a basketball tournament in which the first team to win two games wins the tournament. If the Cardinals have a $\frac{3}{5}$ probability of beating the Ordinals in any particular game, what is the probability that the Ordinals win the tournament?
Answer: $\frac{44}{125}$

B-3. A loaded 6-sided die has its six sides numbered 1 through 6. If the probability of any number being rolled is directly proportional to the number, what is the probability of an odd number being rolled?

Answer: $\frac{3}{7}$

B-4. A set S contains s objects has subsets P and Q containing p and q objects, respectively. If $p < q$, what is the maximum and minimum possible probability of selecting an object from $P \cup Q$ when selecting an object from S at random? Assume all objects in S are equally likely to be selected.

Answer: minimum $= \frac{q}{s}$; maximum $= \frac{p+q}{s}$

C-1. A mother, her daughter, and three intended boys go to sit on a bench. In how many different orders can they sit on the bench so that at most one boy will sit next to the girl?

Answer: 84

C-2. Three cards are selected randomly from a standard deck. What is the probability that no two cards have the same denomination, all three are not of the same suit, and the three cards are not in succession (aces are high)?

Answer: $\frac{44}{1105}$

C-3. The dreaded disease Mathitis affects 5% of the population. A certain test for the disease is successful in detecting its presence in affected people 95% of the time. The same test detects the presence of the disease in unaffected people 4% of the time. If the test detects a person has the disease, what is the probability that this person has the disease?

Answer: $\frac{5}{9}$

C-4. A game at the carnival is played as follows. A contestant is given a disc of radius 6 inches. A five foot square table stands in the center of the booth. The table is divided evenly into two halves, one half colored black and the other half colored white. The contestant tosses the disc, and wins if the disc lies entirely within the white region of the table. What is the probability that disc lying entirely on the table is a winning toss?

Answer: $\frac{3}{8}$

Meet 7

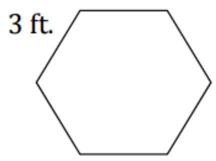
EVENT 2: Geometry – Surface Areas & Volumes of Solids

Include: Ratios and proportions

Forms of Answers Note: Find exact answers (i.e. simplest pi and/or radical form)

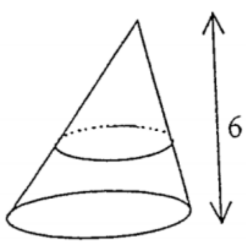
Sample Problems:

A. Dilbert's fishpond is in the shape of a right regular hexagonal prism, top view shown below. The fishpond has a depth of 3 ft. Find the pond's volume.



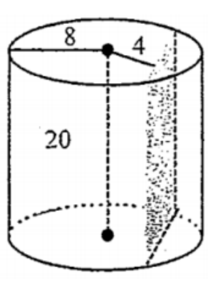
Answer: $\frac{81\sqrt{3}}{2}$ ft^3

B. The radius of the base of a circular cone is 5 and the height is 6. Find the area of the cross section 2 units from the base.



Answer: $\frac{100\pi}{9}$

C. Plane E intersects a right circular cylinder and is parallel to the line that contains the centers of the bases of the cylinder. Find the volume of the smaller spatial region of the cylinder that is cut off by E.



Answer: $\frac{1280\pi}{3} - 320\sqrt{3}$ units^3

Meet 7

EVENT 3: Algebra I – Radical Operations

Include: (1) Variables
(2) Problems dealing with rationalization of expressions where the denominator may have no more than two terms
(3) Equations

Exclude: (1) Simplification which leads to an answer involving absolute value
(2) Conversion of radical expressions to exponential form
(3) Approximation of square roots to a specific decimal place or specific number of significant digits
(4) Radicals with an index ≥ 3

Sample Problems:

A. Express in simplest radical form: $4\sqrt{20}+9\sqrt{45}-4\sqrt{\frac{1}{5}}$

Answer: $\frac{171\sqrt{5}}{5}$

B. Express in simplest radical form, given that $x>0$ and $y>0$: $\sqrt{\frac{4x^{2n}}{9y}}$

Answer: $\frac{2x^{n}\sqrt{y}}{3y}$

C-1. Find the solution set: $\sqrt{5y-9}-y+1=0$

Answer: $\{2,\ 5\}$

C-2. Simplify: $\frac{1+\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

Answer: $-5+2\sqrt{6}+\sqrt{3}-\sqrt{2}$

Meet 7

EVENT 4: Analytic Geometry – Rational Functions (Odd Years)

Notes: (1) The x-intercept is the first coordinate (the number) of the point(s) of intersection of the graph with the x-axis. The y-intercept is the second coordinate of the point(s) of intersection of the graph with the y-axis. If the problem calls for the x- and y-intercepts, to avoid ambiguity, the form of the answer blank should be:

Answer: x-intercept(s) ____________
y-intercept(s) ____________

and the appropriate number(s) should be listed. If there is no x-intercept and/or y-intercepts, write "none".

(2) Asymptotes are lines and should be described by the appropriate linear equation (first degree in x and/or y).

(3) Set or interval notation acceptable for domain and range.

Include: (1) x-intercepts and y-intercepts
(2) Vertical, horizontal, and slant asymptotes
(3) Domain and range of the function
(4) Symmetry with respect to the y-axis and origin

Sample Problems:

A. Find the x-intercept(s) for $y = \dfrac{x^2 - 4}{x^2 - 25}$.

Answer: $2, -2$

B. Find the asymptotes for $y = \dfrac{x^3}{x^2 + x - 12}$.

Answer: vertical: $x = 3$, $x = -4$
horizontal: none
slant: $y = x - 1$

C. Determine the domain and range of $\left\{ (x, y) : y = \dfrac{4x^2 + 1}{x^2 - 1} \right\}$.

Answer: domain: $\{x : x \neq \pm 1\}$
range: $\{y : y \leq -1 \text{ or } y > 4\}$

Meet 7

EVENT 4: Analytic Geometry – Vectors
(Even Years)

Note: In 2D, either $\langle a,\ b\rangle$ or $a\hat{i}+b\hat{j}$ is acceptable; in 3D, either $\langle a,\ b,\ c\rangle$ or $a\hat{i}+b\hat{j}+c\hat{k}$ is acceptable. Student solutions must include arrows to represent a vector: $\vec{u}$ (not u). For unit vectors, students have the option of using an arrow, $\vec{i}$, or a hat, $\hat{i}$. Typeset questions may have arrows, $\vec{u}=\langle 2,\ -5\rangle$, or be in bold, $\mathbf{u}=2\mathbf{i}-5\mathbf{j}$.

Include: (1) Operations with vectors, geometrically and algebraically
(2) Parallel and perpendicular vectors; dot product operations
(3) Magnitude of vectors in 2D or 3D
(4) Determinants of order 3 (to find cross-products)
(5) Computation of the angle between vectors using trigonometry

Exclude: Solving for angles that require trig tables or a calculator

Sample Problems:

A-1. If $\mathbf{u}=\langle 7,\ -7,\ -7\rangle$, find $|\mathbf{u}|$.
Answer: $7\sqrt{3}$

A-2. If $\mathbf{u}=\langle x,\ -3,\ 10\rangle$ and $\mathbf{v}=\langle x,\ x,\ -4\rangle$, and $\mathbf{u}\perp\mathbf{v}$, find all possible values of x.
Answer: –5 , 8

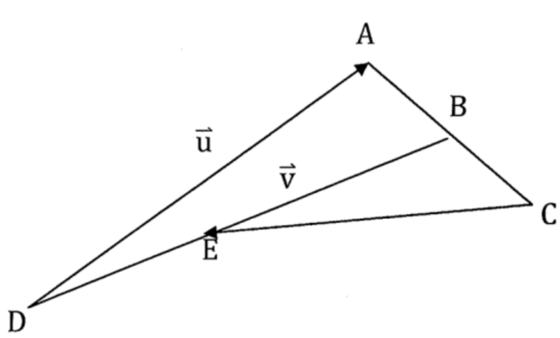
A-3. If $\mathbf{u}=\langle 2,\ 3,\ 4\rangle$, $\mathbf{v}=\langle 6,\ 7,\ 8\rangle$, and $\mathbf{w}=\langle -1,\ 5,\ 9\rangle$, evaluate $\mathbf{w}\cdot(\mathbf{v}\times\mathbf{u})$.
Answer: –8

A-4. Find the cosine of the angle between $\mathbf{a}=\langle 3,\ 4\rangle$ and $\mathbf{b}=\langle 5,\ 12\rangle$.
Answer: 63/65

B-1. Find a vector of length 2 that is perpendicular to both $\langle 5,\ 4,\ 2\rangle$ and $\langle 0,\ 1,\ -2\rangle$.
Answer: $\langle -4/3,\ 4/3,\ 2/3\rangle$ or $\langle 4/3,\ -4/3,\ -2/3\rangle$

B-2. In the figure below, given that $DE:EB=1:2$ and B is the midpoint of $\overline{AC}$. If $\vec{v}=\overline{BE}$ and $\vec{u}=\overline{DA}$, express $\overline{CE}$ in terms of $\vec{u}$ and $\vec{v}$.

Answer: $\vec{u}+\frac{5}{2}\vec{v}$



C-1. If $\mathbf{u}=\langle 5,\ 12\rangle$ and $|\mathbf{v}|=\sqrt{3}$, and the acute angle formed by $\mathbf{u}$ and $\mathbf{v}$ is 60°, find $|\mathbf{u}-\mathbf{v}|$.
Answer: $\sqrt{133}$

C-2. At what point does the line passing through (2, 3, 5) and (8, –17, –3) intersect the plane $4x+3y+2z=1$?
Answer: (5, –7, 1)

Meet 7

EVENT 5: Algebra I – Equations with Decimals & Percents

Include: (1) Investment problems
(2) Interest Problems
(3) Problems regarding mixture of different priced goods
(4) Mixture of substances of different concentrations
(5) Mixture of different priced goods to create a product to sell to make a profit
(6) Problems solved by any of the following methods:
(a) linear systems in one variable
(b) systems of equations in two variables
(c) quadratic equations that are factorable
(7) Explicit instructions regarding unit of measure of the answer (i.e. miles per hour versus miles per minute, seconds versus minutes, hours versus a time of day, miles versus feet, etc.)

Suggestion: Write the unit of measure on the answer blank if only the numerical value is to be graded.

Sample Problems:

A. A coffee merchant blended coffee worth 95 cents per pound with coffee worth $1.20 per pound. The mixture of 30 pounds was valued by the merchant at $1.02 per pound. How many pounds of the cheaper coffee was used?
Answer: 21.6 pounds

B. Solve for x and y:

$$0.3x + 0.4y = 3.4$$
$$0.5x + 0.3y = 1.8$$

Answer: $x = -\frac{30}{11}$, $y = \frac{116}{11}$

C. Two wholesalers, A and B, sell identical products for the prices and discounts given below:
A: $750 – 20% discount – additional 2% discount on the discounted price for paying cash.
B: $800 – 25% discount – additional 5% discount on the discounted price for paying cash.
What is the amount of difference between the two offers?
Answer: $18

Meet 7

EVENT 6: Calculus – Integrals

Include: (1) Computing definite and indefinite integrals
(2) Computing integrals by substitution
(3) Solving differential equations by separating variables
(4) Computing areas
(5) Computing volumes (using slices, disks, washers)
(6) Displacement & total distance traveled by a particle moving on a number line
(7) Average value
(8) The Fundamental Theorems of Calculus
(9) Using Riemann sums and trapezoids to estimate definite integrals
(10) Slope fields (no drawing)

Exclude: (1) Integration by parts
(2) Trigonometric substitution
(3) Partial fractions
(4) Computing volumes using shells
(5) Computing lengths of curve, surface area, and centroids
(7) Trapezoidal error
(8) Simpson's Rule
(9) Parametric & polar functions
(1) Improper integrals

Sample Problems:

A-1. Compute: $\int \frac{x+1}{x}\,dx$

Answer: $x+\ln|x|+C$

A-2. Compute: $\int_0^{\ln 3} e^{2x}\,dx$

Answer: 4

A-3. Compute: $\int_0^{\pi/2} \frac{\cos x\,dx}{\sqrt{1+3\sin x}}$

Answer: 2/3

A-4. Compute: $\int_1^{e} \frac{\ln x}{x}\,dx$

Answer: 1/2

B-1. A particle moves on a number line with velocity $v(t)=3t^2-12t+9$. From $t=0$ to $t=5$, find the total distance traveled by the particle.
Answer: 28

B-2. Find the area of the region bounded by $y=\sqrt{x}$ and $y=\frac{x^2}{8}$.

Answer: 8/3

B-3. Water is flowing into a tank at a rate of $\frac{t}{t^2+1}$ gallons per minute (t measured in minutes). If the tank is empty to begin with, how many gallons will it contain at the end of 10 minutes?

Answer: $\frac{\ln 101}{2}$

B-4. Solve the differential equation $\frac{dy}{dx}=\frac{2x}{y^2}$ with initial condition $y(1)=3$.

Answer: $y=\sqrt[3]{3x^2+24}$

B-5. f is a continuous function with values as shown below. Estimate $\int_{-1}^{17} f(x)\,dx$ using the midpoint of 3 rectangles.

x	-1	2	5	8	11	14	17
$f(x)$	1	2	3	5	8	13	21

Answer: 120

B-6. The velocity (v) of a particle moving on a number line is a continuous function with values as shown below. Estimate the distance traveled from $t=1$ to $t=10$ using 3 trapezoids.

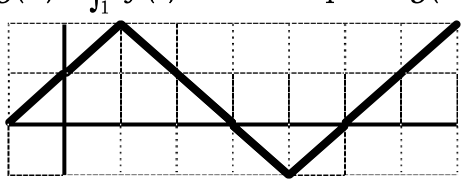
t	1	3	6	10
$v(t)$	7	5	3	2

Answer: 34

B-7. Compute the average value of $f(x)=\sin 2x$ from $x=0$ to $x=\dfrac{\pi}{2}$.

Answer: $\dfrac{2}{\pi}$

B-8. Given the graph of $y=f(x)$ defined on $-1 \le x \le 7$ as shown below, and $g(x)=\int_1^x f(t)\,dt$. Compute $g(-1)$.



Answer: -2

B-9. The base of a solid is the region bounded by $y=\sqrt{x}$, $y=2$ and the y-axis. Compute the volume of the solid if cross sections perpendicular to the y-axis are squares.

Answer: $\dfrac{32}{5}$

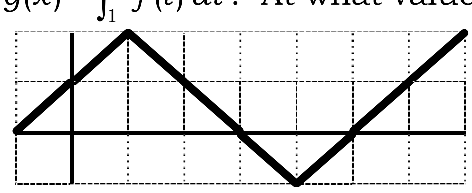
C-1. R is the region bounded by $y=\sqrt{x}$, $y=2$ and the y-axis. Compute volume of the solid generated when R is revolved about the line $y=2$.

Answer: $\dfrac{8\pi}{3}$

C-2. The region bounded by $y=1/x^2$, $y=0$, $x=1$ and $x=4$ is divided into 2 regions of equal area by the vertical line $x=k$. Find the value of k.

Answer: $\dfrac{8}{5}$

C-3. Given the graph of $y=f(x)$ defined on $-1 \le x \le 7$ as shown below, and $g(x)=\int_1^x f(t)\,dt$. At what value(s) of x does g have a point of inflection?



Answer: 1, 4