Meet 2

EVENT 4: Trigonometry – Equations & Graphs

Include: (1) Variable will have a restricted domain that will indicate whether the answer should be in degrees or radians.
(2) May include trigonometric graphs

Exclude: (1) Inverses: problems should be solvable without the use of the inverse functions (NO Arcsin, Arccos, etc.)
(2) Sums to products, products to sums

Sample Problems:

A-1. Solve for x such that $0 \le x < 2\pi$: $2\cos^2 x - \cos x = 1$
Answer: $0, \frac{2\pi}{3}, \frac{4\pi}{3}$

A-2. Solve for x such that $0° \le x < 360°$: $\sin 2x = 1$
Answer: $45°, 225°$

B. Solve for x such that $0° \le x < 360°$: $\sin x - \sqrt{3}\cos x = 1$
Answer: $90°, 210°$

C. Solve for x such that $0° \le x < 360°$: $2\cos^2 \frac{x}{2} = \sin^2 x$
Answer: $90°, 180°, 270°$

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations and Graphs

October 2017

A. Solve for θ, $0 \le \theta < 2\pi$: $4\cos^2\theta = 1$

(2 pts)

ANSWER: $\theta =$ _____

B. Solve for θ, $0 \le \theta < 2\pi$: $\tan\left(\frac{\theta}{2} + \frac{\pi}{3}\right) = 1$

(3 pts)

ANSWER: $\theta =$ _____

C. Solve for θ, $0 \le \theta < 2\pi$: $(2\cos\theta + 1)(1 - \cos\theta) = \cos\theta(1 - 2\sin\theta)$

(5 pts)

ANSWER: $\theta =$ _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number of Solutions and Graphs

October 2016

A. Solve for x such that $0 \le x < \pi$: $4\cos(-x) = -\csc x$

(2 pts)

ANSWER: $x =$ _____

B. Solve for x such that $-2\pi \le x \le 2\pi$: $\cos^2\left(\frac{x}{2}\right) = \sin^2\left(\frac{x}{2}\right) - 1$

(3 pts)

ANSWER: $x =$ _____

C. Solve for x such that $-\frac{\pi}{2} \le x < \frac{\pi}{2}$: $4\sin^2 x + \cot^2 x - 4\cos^2 x = 1$

(5 pts)

ANSWER: $x =$ _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number of Solutions and Graphs

October 2015

A. Solve for θ, $0 \le \theta < 2\pi$: $\sin^2\theta + \cos\theta = -1$

(2 pts)

ANSWER: $\theta =$ _____

B. Solve for θ, $0 \le \theta \le 2\pi$: $2^{\frac{3}{2}} \sin\theta\cos\theta = 1^{\frac{3}{2}}$

(3 pts)

ANSWER: $\theta =$ _____

C. For $\frac{\pi}{2} < x < \frac{3\pi}{2}$, find the x-intercept(s) of the graph of $y = 2\sin\left(x + \frac{\pi}{2}\right) + 3\tan(\pi - x)$.

(5 pts)

ANSWER: _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasoanable Number of Solutions and Graphs

October 2014

A. Find all θ such that $\cos(-\theta) = \sin(-\theta)$ and $0 < \theta < 2\pi$.

(2 pts)

ANSWER: $\theta =$ _____

B. Solve for x, $0 \le x < 2\pi$: $-\cot x + \tan\frac{x}{2} = \sin x$

(3 pts)

ANSWER: $x =$ _____

C. Solve for x, $0 \le x < 2\pi$: $\sqrt{\frac{6\tan(x)}{1-\tan^2(x)} + 1} = 2$

(5 pts)

ANSWER: $x =$ _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number of Solutions and Graphs

October 2013

A. Solve for x such that $0 \le x < 2\pi$: $\cos^2 x = \sin^2 x + \frac{1}{2}$

(2 pts)

ANSWER: $x =$ _____

B. Solve for x where $0 \le x < 2\pi$: $\cos^2 2x = \cos 2x + \sin^2 2x$

(3 pts)

ANSWER: $x =$ _____

C. Solve for x: $\sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \csc^2 x = 7$, $0 \le x \le \pi$

(5 pts)

ANSWER: $x =$ _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number of Solutions and Graphs

October 2012

A. Solve for θ such that $0° \le \theta < 180°$: $\sqrt{3}\tan(\theta - 10°) = -1$

(2 pts)

ANSWER $\theta =$ _____

B. Solve for x such that $0° \le x \le 360°$, $\sec x \tan x - 2\tan x - 2 + \sec x = 0$

(3 pts)

ANSWER: $x =$ _____

C. Solve for x such that $0 \le x < 2\pi$: $\dfrac{\sin(3x)}{\sin x} = 2\sin^2 x$

(5 pts)

ANSWER: $x =$ _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number of Solutions and Graphs 2011

A. Solve for x such that $2\sin 4x - \sqrt{3} = 0$, $0 \le x < \frac{\pi}{2}$

(2 pts)

ANSWER: _____

B. At what values of x $(-12 < x < 12)$ does the graph of $y = 2\sec\left[\frac{\pi}{6}(x-1)\right] + 3$ have

(3 pts) a vertical asymptote?

ANSWER: _____

C. Solve for θ, $0° \le \theta < 360°$,

(5 pts) $\cos 311°\cos 2\theta - \cos 311°\sin 2\theta = \sin 311°\cos 2\theta + \sin 311°\sin 2\theta$

ANSWER: _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number of Solutions & Graphs 2010

A. Solve for x such that $0 \le x < 2\pi$: $\cos x = -\sqrt{3}\sin x$

(2 pts)

ANSWER: $x =$ _____

B. Solve for x such that $0 \le x < 2\pi$: $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$

(3 pts)

ANSWER: $x =$ _____

C. Solve for φ, $0 \le \varphi < 2\pi$: $\tan\left(\varphi + \frac{5\pi}{4}\right) - \cos^2(\varphi + \pi) + \sin^2(\varphi + \pi) = 0$

(5 pts)

ANSWER: $\varphi =$ _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number of Solutions 2009

A. (2 pts) Solve for θ such that $0^\circ \le \theta < 180^\circ$

$$4\cos^2 2\theta = 3$$

ANSWER: _____

B. (3 pts) Solve for θ where $0^\circ \le \theta < 360^\circ$

$$2\cos^2 \theta + \sin\theta = 1$$

ANSWER: _____

C. (5 pts) Solve for x where $0 \le x < 2\pi$

$$\sec^2 x - 2\csc x + 2\sec x - \cot x \sec^2 x + 3 = \tan^2 x$$

ANSWER: _____

Name _____ Score _____ School _____

Event 4: TRIGONOMETRY — Equations with a Reasonable Number sol Solutions and Graphs 7008

A. (2 pts) Solve for θ, $0° < \theta \leq 180°$: $\dfrac{\tan 85° - \tan\theta}{1 + \tan 85° \tan\theta} = 1$

ANSWER: _____

B. (3 pts) Solve for x, $0 \leq x < 2\pi$: $\sin x\left(\cot^2 x\right) = 3\sin x$

ANSWER: _____

C. (5 pts) Solve for x, $\pi \leq x < 2\pi$: $\sin(4x) = \sin(2x)$

ANSWER: _____

Name_____ Score_____ School_____

Event 4: TRIGONOMETRY – Equations with a Reasonable Number of Solutions 2007

A. (2 pts) Solve for x:

$\cos x \cot x = \cos x,\ 0 \le x < 2\pi$

ANSWER $x =$ _____

B. (3 pts) Solve for x if $0° \le x \le 180°$:

$\cos x + \sin x + \sin 2x = -\frac{1}{2}$

ANSWER $x =$ _____

C. (5 pts) Solve for x:

$0.5\cot(2x) - \cos^2 x + \sin^2 x = 0,\ \pi \le x < 2\pi$

ANSWER $x =$ _____

Name_____ Score_____ School_____

Event 4: TRIGONOMETRY – Equations with a Reasonable Number of Solutions and Graphs 2006

A. (2 pts) Solve for x, $0 \le x < 2\pi$

$2\cos^4 x = \cos^2 x$

ANSWER _____

B. (3 pts) Solve for θ, $0 \le \theta < 2\pi$

$\sec\theta - \tan\theta = 1$

ANSWER _____

C. (5 pts) Solve for θ, $0 \le \theta < 2\pi$

$$(1 + \tan^2\theta)\sin 2\theta \left(\frac{\tan\pi - \tan\frac{\pi}{6}}{1 + \tan\pi\tan\frac{\pi}{6}}\right) = 2$$

ANSWER _____

Name_____ Score_____ School_____

Event 4: Trigonometry - Equations with a Reasonable Number of Solutions or Graphs 2005

A. (2 pts) Solve for x, $0 \le x < 2\pi$:

$$\cos 2x + \sin x = 0$$

ANSWER: _____

B. (3 pts) Solve for x, $0° \le x < 360°$:

$$3\tan^3 x - \tan x - 3\sec^2 x = -4$$

ANSWER: _____ °

C. (5 pts) Solve for x, $0° \le x < 360°$:

$$2\sqrt{3}\sin^2 \frac{x}{2} - 2\sin x + \sin 2x = 0$$

ANSWER: _____ °

Name_____ _ Score_____ School_____

Event 4: Trigonometry - Equations with a Reasonable Number of Solutions 2004

A. (2 pts) Find the number of solutions to $\tan x = 2004$, for $-2004\pi \le x \le 2004\pi$.

ANSWER: _____

B. (3 pts) Suppose that the length of daylight y hours in Seattle, Washington, varies sinusoidally with the t^{th} day in the calendar year.

If $y = 12.25 + 3.75\cos\left(\frac{2\pi}{365}t - \frac{184\pi}{365}\right)$, find which day, t, Seattle has exactly 16 hours of daylight, where $0 \le t < 365$.

ANSWER: _____

C. (5 pts) Solve for x, $0 \le x < 2\pi$:

$$(2\sin x \sec x + 4\sin x - \sec x - 2) = 0$$

ANSWER: _____

Name_____ Score_____ School_____

Event 3: Trigonometry - Equations with a Reasonable Number of Solutions 2003

A. (2 pts) A coil of wire rotating in a magnetic field induces a voltage e given by

$$e = 20\sin\left(\frac{\pi}{4}t - \frac{\pi}{2}\right),$$ where t is time in seconds.

Find the smallest positive time to produce a voltage of $10\sqrt{3}$.

ANSWER: _____ seconds

B. (3 pts) Solve for x, $0 \le x < 2\pi$:

$$\tan x \sin x - \sin x - \tan x + 1 = 0.$$

ANSWER: _____

C. (5 pts) Solve for x, $0 \le x < 2\pi$:

$$4\sin^2\left(\frac{x}{2}\right) = 3\tan^2\left(\frac{x}{2}\right) - 1$$

ANSWER: _____

Name_____ Score_____ School_____

Event 3; Trigonometry - Equations with a Reasonable Number of Solutions 2002

A. (2 pts) Solve for x such that $-\pi \le x < \pi$:

$$\sin\left(x - \tfrac{\pi}{5}\right) = -1$$

ANSWER: _____

B. (3 pts) Solve for θ such that $0 \le \theta < 2\pi$:

$$\cos\theta = \frac{\sin^2\theta + \sin\theta \cdot \cos\theta + 1}{\sin\theta + \cos\theta}$$

ANSWER: _____

C. (5 pts) Solve for θ such that $0 \le \theta < 2\pi$:

$$\tan^2\frac{\theta}{2}(1+\cos\theta) - \sin\theta = 0$$

ANSWER: _____

Name_____ Score_____ School_____

Event 3: Trigonometry - Equations with a Reasonable Number of Solutions 2001

A. (2 pts) Solve for x where $0 \le x < 2\pi$:

$$\tan x = \tan^2 x$$

ANSWER: _____

B. (3 pts) Solve for x such that $0° \le x < 360°$:

$$\tan x \csc x - \csc x - 2\tan x + 2 = 0$$

ANSWER: _____

C. (5 pts) Solve for x where $0 \le x < 2\pi$:

$$\sqrt{4\sin 2x + 7} - 3 = 0$$

ANSWER: _____

Name _____ Score _____ School _____

Event 3: TRIGONOMETRY - Equations with a Reasonable Number of Solutions 2000

A. (2 pts) Solve for x, $0° \le x < 360°$:

$$\csc^2 x - \csc x - 2 = 0.$$

ANSWER _____

B. (3 pts) Solve for x, $0 \le x < 2\pi$:

$$\tan(x+\pi) - \cos(x+\frac{\pi}{2}) = 0$$

ANSWER _____

C. (5 pts) Solve for x, $0 \le x < 2\pi$:

$$\sin 4x + \sin 2x = -2 \cos x$$

ANSWER _____

Name_____ Score_____ School_____

Event 3: Trigonometry - Equations With a Reasonable Number of Solutions 1999

A. (2 pts) Solve for α such that $0° \le \alpha < 360°$.

$$\frac{\tan\alpha - \tan 40°}{1 + \tan\alpha \tan 40°} = \sqrt{3}$$

ANSWER: _____

B. (3 pts) Solve for x such that $0 \le x < 2\pi$.

$$\sin x + \cos x = 1$$

ANSWER: _____

C. (5 pts) Solve for x such that $0 \le x < 2\pi$.

$$\tan 2x + 2\sin x = 0$$

ANSWER: _____

Name_____ Score_____ School_____

Event 3: Trigonometry - Equations with a Reasonable Number of Solutions 1998

A. (2 pts) Solve for x such that $0° \le x < 360°$.

$$\cos 2x \sec x = 2\cos 2x$$

ANSWER: _____

B. (3 pts) Solve for x such that $0 \le x < 2\pi$.

$$\sin 2x = \cot x - \cos x$$

ANSWER: _____

C. (5 pts) Solve for x such that $0 \le x < \pi$.

$$\sin(2\pi x) + \sin(\pi x) = 0$$

ANSWER: _____

Name _____ Score _____ School _____

Event 3: TRIGONOMETRY - Equations with a reasonable number of solutions 1997

A. (2 pts) Solve for x such that $0 \leq x < 360^\circ$:

$$\cos 2x = \cos x$$

ANSWER _____

B. (3 pts) Solve for x such that $0 \leq x < 360^\circ$:

$$\sin 2x + \sqrt{2} \cos x = 0$$

ANSWER _____

C. (5 pts) Solve for x such that $0 \leq x < 2\pi$:

$$\sqrt{2} (\sin x + \cos x) = \sqrt{3}$$

ANSWER _____

Name _____ Score _____ School _____

Event 3: TRIGONOMETRY - Equations with a reasonable number of solutions 1996

A. (2 pts) Solve for x such that $0 \le x < 2\pi$: $2\sin^2 x + \sin x = 1$

ANSWER _____

B. (3 pts) Solve for x such that $0 \le x < 2\pi$: $\dfrac{\cos x}{1 - \sin x} = 1$

ANSWER _____

C. (5 pts) Solve for θ such that $0^\circ \le \theta < 360^\circ$: $2\cos 3\theta + \tan 3\theta - \sec 3\theta = 0$

ANSWER _____

Name _____ Score _____ School _____

Event 3: TRIGONOMETRY - Equations with a reasonable number of solutions and graphs 1995

A. (2 pts) If $0^\circ \le x < 360^\circ$, solve for x: $2 \sin x \sec x = \sec x$

ANSWER _____

B. (3 pts) If $0 \le x < 2\pi$, solve for x: $3\cos x = 2\sin^2 x$

ANSWER _____

C. (5 pts) If $0^\circ \le x < 360^\circ$, solve for x: $1 + \tan(110^\circ + 3x)\tan(20^\circ + 5x) = \tan(110^\circ + 3x) - \tan(20^\circ + 5x)$

ANSWER _____

Name _____ Score _____ School _____

Event 3: TRIGONOMETRY - Equations with a reasonable number of solutions 1994

A. (2 pts) Solve for x, $0 \le x < 2\pi$:

$$\sin x + \cos x = 1$$

ANSWER _____

B. (3 pts) Solve for x, $0 \le x < 2\pi$:

$$4 \sin x \cos x = -\sqrt{2}$$

ANSWER _____

C. (5 pts) Solve for x, $0 \le x < 2\pi$:

$$\cos 3x + \cos x = 0$$

ANSWER _____

Name _____ Score _____ School _____

Event 3: TRIGONOMETRY - Equations with a reasonable number of solutions 1993

A. (2 pts) Solve for x:

$$\tan x = 1,\ 6\pi \le x < 8\pi$$

ANSWER _____

B. (3 pts) Solve for θ:

$$\frac{1}{\cos\theta + \sin\theta} + \frac{1}{\cos\theta - \sin\theta} = 2,\quad 0^\circ \le \theta < 360^\circ$$

ANSWER _____

C. (5 pts) Solve for x:

$$2\cos 5x \cos 3x = 1 - 2\sin 5x \sin 3x,\quad 0 \le x < 2\pi$$

ANSWER _____

Meet 2, Event 4: TRIGONOMETRY
Equations & Graphs

2017

A. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
B. $\frac{11\pi}{6}$
C. $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

2016

A. $\frac{7\pi}{12}, \frac{11\pi}{12}$
B. $-\pi, \pi$
C. $\pm\frac{\pi}{6}, \pm\frac{\pi}{4}$

2015

A. π
B. $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$
C. $\frac{5\pi}{6}$

2014

A. $\frac{3\pi}{4}, \frac{7\pi}{4}$
B. $\frac{\pi}{2}, \frac{3\pi}{2}$
C. $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

2013

A. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
B. $0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$
C. $\frac{\pi}{4}, \frac{3\pi}{4}$

2012

A. 160°
B. 60°, 135°, 300°, 315°
C. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

2011

A. $\frac{\pi}{12}, \frac{\pi}{6}$
B. –8, –2, 4, 10
C. 47°, 137°, 227°, 317°

2010

A. $\frac{5\pi}{6}, \frac{11\pi}{6}$
B. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{7\pi}{6}, \frac{11\pi}{6}$
C. $0, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{3\pi}{2}, \frac{7\pi}{4}$

2009

A. 15°, 75°, 105°, 165°
B. 90°, 210°, 330°
C. $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}$

2008

A. 40°
B. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
C. $\pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

2007

A. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{4}$
B. 120°
C. $\frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}$

2006

A. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$
B. 0
C. $\frac{2\pi}{3}, \frac{5\pi}{3}$

2005

A. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
B. 30°, 45°, 150°, 210°, 225°, 330°
C. 0°, 60°, 120°

2004

A. 4008 solutions
B. 92
C. $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}$

Note: Answers are shown as they appear on the original answer keys. There may be inconsistencies with the formatting of these answers. In all cases, consult the Guidelines for Forms of Answers to determine the correct formatting.

2003

A. 10/3 seconds

B. $\frac{\pi}{4}, \frac{5\pi}{4}$

C. $\frac{\pi}{2}, \frac{3\pi}{2}$

2002

A. $-\frac{3\pi}{10}$

B. $0, \pi$

C. $0, \frac{\pi}{2}$

2001

A. $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

B. 30°, 45°, 150°, 225°

C. $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

2000

A. 30°, 150°, 270°

B. $0, \pi$

C. $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

1999

A. 100°, 280°

B. $0, \frac{\pi}{2}$

C. $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

1998

A. 45°, 60°, 135°, 225°, 300°, 315°

B. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

C. $0, \frac{2}{3}, 1, \frac{4}{3}, 2, \frac{8}{3}, 3$

1997

A. 0°, 120°, 240°

B. 90°, 225°, 270°, 315°

C. $\frac{\pi}{12}, \frac{5\pi}{12}$

1996

A. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

B. 0

C. 70°, 110°, 190°, 230°, 310°, 350°

1995

A. 30°, 150°

B. $\frac{\pi}{3}, \frac{5\pi}{3}$

C. 22.5°, 112.5°, 202.5°, 292.5°

1994

A. $0, \frac{\pi}{2}$

B. $\frac{5\pi}{8}, \frac{7\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

C. $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

1993

A. $\frac{25\pi}{4}, \frac{29\pi}{4}$

B. 0°, 120°, 240°

C. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

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