

Meet 4

EVENT 4: Calculus – Limits and Derivatives (odd-even years)

- Include:
- (1) Computing limits
 - (2) Continuity & differentiability
 - (3) Average rates of change
 - (4) Computing derivatives
 - (5) Implicit differentiation
 - (6) Applications of derivatives: optimization, related rates, linearization, particle motion
 - (7) Curve analysis: increasing, decreasing, extrema, concavity, inflection points
 - (8) The Mean Value Theorem
 - (9) Average rate of change

- Exclude:
- (1) Problems where the answer is ∞ or $-\infty$
 - (2) Indeterminate forms that are not $0/0$ or ∞/∞
 - (3) Parametric & polar functions

Note: (1) When a limit does not exist, “does not exist,” or “non-existent,” are acceptable answers, “DNE” is not acceptable.

Sample Problems:

A-1. Compute: $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$

Answer: 3

A-3. Compute: $\lim_{x \rightarrow \infty} \sin^{-1}\left(\frac{x+3}{2x+5}\right)$

Answer: $\pi/6$

A-2. Compute: $\lim_{x \rightarrow 0} \frac{x + \sin x}{x}$

Answer: 2

A-4. Compute: $\lim_{x \rightarrow \infty} \frac{x^2}{2^x}$

Answer: 0

A-5. Compute: $\lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - \tan\frac{\pi}{4}}{h}$

Answer: 2

A-6. Find the average rate of change of $f(x) = \sqrt{x}$ from $x = 1$ to $x = 9$.

Answer: $\frac{1}{4}$

B-1. Compute: $\frac{d}{dx}(-2x \cos 2x + \sin 2x)$

Answer: $4x \sin 2x$

B-2. Given the function $f(x) = \sqrt{x^2 + 9}$, use the linearization of $f(x)$ at $x = 4$ to estimate $f(4.2)$.

Answer: 5.16

B-3. If $f(x) = xe^{-x}$, on what interval(s) is f concave up?

Answer: $x > 2$

B-4. Given the curve $x^2 + xy + y^3 = 7$, find the slope of the tangent line at the point $(2, 1)$.

Answer: -1

B-5. Let $f(x) = \begin{cases} ax^2 + bx, & x < 1 \\ x^3, & x \geq 1 \end{cases}$

Find the values of a and b so that f will be continuous and differentiable at $x = 1$.

Answer: $a = 2, b = -1$

B-6. Find the value(s) of c that satisfy the conclusion of the Mean Value Theorem for $f(x) = \sqrt{x}$ on the interval $[1, 9]$.

Answer: 4

C-1. A right circular cone is formed by revolving a right triangle with hypotenuse 2 cm about one of its legs. If x represents the height of the cone, find the value of x (in cm) that will maximize the volume of the cone.

Answer: $\frac{2\sqrt{3}}{3}$

C-2. If $y = \frac{x}{x^2 + 1}$, for what values of x is $\frac{d^2y}{dx^2} = 0$?

Answer: $0, \pm\sqrt{3}$

C-3. A particle's position on the x -axis is given by $x(t) = t^3 - 6t^2 + 9t - 4$ for $t \geq 0$. For what values of t is the particle speeding up?

Answer: $1 < t < 2$ or $t > 3$

C-4. The length of a rectangle is increasing at a rate of 3 inches per minute and the width is decreasing at a rate of 2 inches per minute. At the moment when the length is 8 inches and the width is 6 inches, how fast is the angle formed by the length and the diagonal changing? Give answer in radians per minute.

Answer: $-\frac{17}{50}$

Name _____ Score _____ School _____

Event 4: CALCULUS — Limits and Derivatives

January 2020

A. Evaluate: $\lim_{x \rightarrow 0} \frac{x}{x + \ln(x + 1)}$

(2 pts)

ANSWER: _____

B. Let $f(x) = -x^2 + 8x - 8 + e^x$. Find all values of x at which the graph of f has a point of inflection.

(3 pts)

ANSWER: _____

C. A trash compactor is in the shape of a rectangular solid with height 10 meters. Its width is decreasing at the rate of $\frac{1}{3}$ meter per second, and its length is increasing at the rate of $\frac{1}{2}$ meter per second. When the volume is 30 cubic meters and the width is 1 meter, find the rate at which the volume is decreasing.

(5 pts)

ANSWER: _____ cubic meters per second

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Event 6: CALCULUS — Limits and Derivatives

January 2019

- A. Compute: $\lim_{x \rightarrow 0} \frac{3\sin x + x \sin x}{4x^2 - 36x}$
(2 pts)

ANSWER: _____

- B. The curve $y = e^x$ has a tangent line that is parallel to the line $y = 2x - 3$. Find the equation of that tangent line. Write your answer in slope-intercept form.
(3 pts)

ANSWER: _____

- C. Find the slope of the line tangent to $y = \lim_{h \rightarrow 0} \frac{\sec^2(x+h) - \sec^2 x}{h}$ at $x = \frac{\pi}{6}$.
(5 pts)

ANSWER: _____

Name _____ Score _____ School _____

Event 6: CALCULUS — Limits and Derivatives

January 2018

A. Compute: $\lim_{x \rightarrow 0} \frac{\sin x - 5x}{x}$

(2 pts)

ANSWER: _____

B. Given the curve $xy^2 - x^3y = 6$. Find the slope of the line tangent to the curve at $(-2, 1)$.

(3 pts)

ANSWER: _____

C. A spotlight on the ground must follow a New Year's Eve ball as it drops to the ground. The spotlight is 40 feet from the landing site of the ball, and the ball is dropping at a rate of 2 ft/sec. When the ball is 30 feet in the air, find the rate at which the angle formed by the beam of light and the ground is decreasing. Give your answer in radians per second.

(5 pts)

ANSWER: _____ radians/second

Meet 4, Event 4: CALCULUS – Limits & Derivatives

2020

A. $1/2$

B. $\ln 2$

C. 5

2019

A. $-1/12$

B. $y = 2x - 2 \ln 2 + 2$

C. $16/3$

2018

A. -4

B. $11/4$

C. $4/125$