Meet 7

EVENT 4: Analytic Geometry – Vectors (odd-even years)

Note: In 2D, either $\langle a, b\rangle$ or $a\hat{i}+b\hat{j}$ is acceptable; in 3D, either $\langle a, b, c\rangle$ or $a\hat{i}+b\hat{j}+c\hat{k}$ is acceptable. Student solutions must include arrows to represent a vector: $\vec{u}$ (not u). For unit vectors, students have the option of using an arrow, $\vec{i}$, or a hat, $\hat{i}$. Typeset questions may have arrows, $\vec{u}=\langle 2, -5\rangle$, or be in bold, $\mathbf{u}=2\mathbf{i}-5\mathbf{j}$.

Include: (1) Operations with vectors, geometrically and algebraically
(2) Parallel and perpendicular vectors; dot product operations
(3) Magnitude of vectors in 2D or 3D
(4) Determinants of order 3 (to find cross-products)
(5) Computation of the angle between vectors using trigonometry

Exclude: Solving for angles that require trig tables or a calculator

Sample Problems:

A-1. If $\mathbf{u}=\langle 7, -7, -7\rangle$, find $|\mathbf{u}|$.
Answer: $7\sqrt{3}$

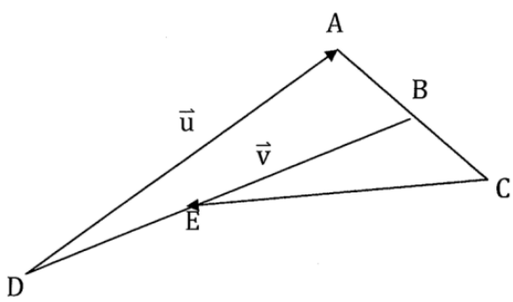
A-2. If $\mathbf{u}=\langle x, -3, 10\rangle$ and $\mathbf{v}=\langle x, x, -4\rangle$, and $\mathbf{u}\perp\mathbf{v}$, find all possible values of x.
Answer: -5, 8

A-3. If $\mathbf{u}=\langle 2, 3, 4\rangle$, $\mathbf{v}=\langle 6, 7, 8\rangle$, and $\mathbf{w}=\langle -1, 5, 9\rangle$, evaluate $\mathbf{w}\cdot(\mathbf{v}\times\mathbf{u})$.
Answer: -8

A-4. Find the cosine of the angle between $\mathbf{a}=\langle 3, 4\rangle$ and $\mathbf{b}=\langle 5, 12\rangle$.
Answer: $\frac{63}{65}$

B-1. Find a vector of length 2 that is perpendicular to both $\langle 5, 4, 2\rangle$ and $\langle 0, 1, -2\rangle$.
Answer: $\left\langle -\frac{4}{3}, \frac{4}{3}, \frac{2}{3}\right\rangle$ or $\left\langle \frac{4}{3}, -\frac{4}{3}, -\frac{2}{3}\right\rangle$

B-2. In the figure below, given that $DE:EB=1:2$ and B is the midpoint of $\overline{AC}$. If $\vec{v}=\overrightarrow{BE}$ and $\vec{u}=\overrightarrow{DA}$, express $\overrightarrow{CE}$ in terms of $\vec{u}$ and $\vec{v}$.
Answer: $\vec{u}+\frac{5}{2}\vec{v}$



C. If $\mathbf{u}=\langle 5, 12\rangle$ and $|\mathbf{v}|=\sqrt{3}$, and the angle formed by $\mathbf{u}$ and $\mathbf{v}$ is $30°$, find $|\mathbf{u}-\mathbf{v}|$.
Answer: $\sqrt{133}$

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Vectors

April 2019

A. (2 pts) If $\vec{u} = \langle -3, 1, 2 \rangle$, $\vec{v} = \langle 0, 2, 3 \rangle$ and $\vec{w} = \langle 2, -1, -2 \rangle$, find $2\vec{u} + \vec{v} - \vec{w}$.

ANSWER: _____

B. (3 pts) θ is the angle between $\vec{u} = \vec{i} + 2\vec{j} + 2\vec{k}$ and $\vec{v} = -\vec{i} + \vec{j} + \vec{k}$. Find $\cos\theta$.

ANSWER: _____

C. (5 pts) Evaluate $\vec{v}_1 \bullet \left(\vec{v}_2 \times \vec{v}_3 \right)$ where $\vec{v}_1 = \langle 4, -5, -1 \rangle$, $\vec{v}_2 = \langle 1, 2, -2 \rangle$, and $\vec{v}_3 = \langle 2, 0, -1 \rangle$.

ANSWER: _____

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY – Vectors in Two and Three Dimensions

April 2017

A. (2 pts) Given A(–1, 1, 5) and B(2, 5, 0). Find the unit vector in the direction of $\overrightarrow{AB}$.

ANSWER: $\langle \quad , \quad , \quad \rangle$

B. (3 pts) If $\mathbf{u} = \langle a, 1, 2 \rangle$ and $\mathbf{v} = \langle 1, 2, a \rangle$, find a so that $2\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - 2\mathbf{v}$ are perpendicular.

ANSWER: $a =$ _____

C. (5 pts) Given A(0, 0, 1), B(2, 0, 0), and C(0, 3, 0). Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

ANSWER: $\langle \quad , \quad , \quad \rangle$

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Vectors

April 2015

A. (2 pts) Let $\vec{u} = 2\vec{i} + a\vec{j} + \vec{k}$ and $\vec{v} = 4\vec{i} - 2\vec{j} - 2\vec{k}$. Find the value of *a* so that $\vec{u}$ and $\vec{v}$ are perpendicular.

ANSWER: _____ $a =$ _____

B. (3 pts) Find the angle θ, $0° < \theta < 180°$ formed by the vectors $\vec{u} = \langle 2, 1 \rangle$ and $\vec{v} = \langle -1, -3 \rangle$.

ANSWER: _____ °

C. (5 pts) Evaluate: $\left(2\vec{i} - 3\vec{j}\right) \bullet \left[\left(\vec{i} + \vec{j} - \vec{k}\right) \times \left(3\vec{i} - \vec{k}\right)\right]$

ANSWER: _____

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Vectors

April 2013

A. (2 pts) Find the vector with magnitude 39 units that is opposite to the vector $\vec{v} = 5\vec{i} - 12\vec{j}$.

ANSWER: _____

B. (3 pts) Given that $\vec{u} = \langle 1, 2 \rangle$ and $\vec{v} = \langle 1, -3 \rangle$, find the angle θ between the vectors, $0° < \theta < 180°$.

ANSWER: _____°

C. (5 pts) If $\vec{w} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{v} = \langle 1, 1, 3 \rangle$ find the magnitude of $\vec{v} \times \vec{w}$.

ANSWER: _____

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Vectors 2011

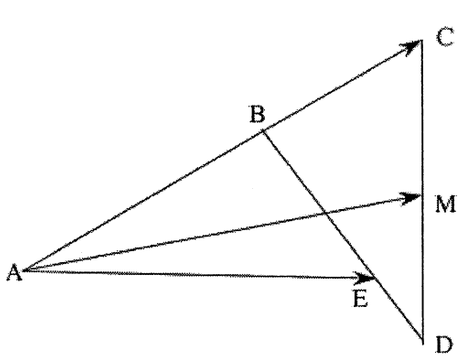
A. (2 pts) If $\vec{u} = \langle 2, 3 \rangle$, find all vectors $\vec{v}$ so that $\vec{u}$ and $\vec{v}$ are perpendicular and have the same magnitude.

ANSWER: _____

B. (3 pts) Given P(3, –2), I(1, 4), and N(2, –5). Find the coordinates of K if $\overrightarrow{PK} = \frac{2}{3}\overrightarrow{PI} - \frac{1}{3}\overrightarrow{PN}$.

ANSWER: _____

C. (5 pts) In the figure below, AB : BC = 3 : 2, BE : ED = 3 : 1, and M is equidistant from C and D. If $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{AE}$, find $\overrightarrow{AM}$ in terms of $\vec{u}$ and $\vec{v}$.



ANSWER: _____

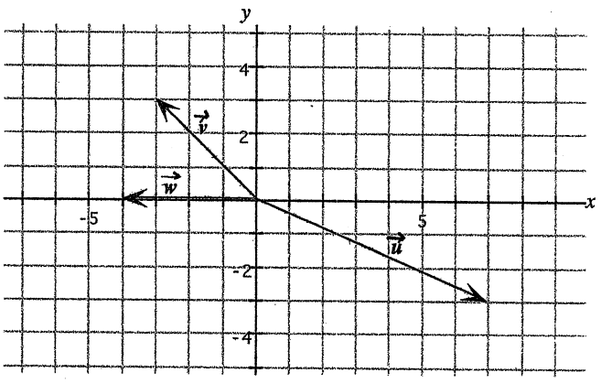
Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Vectors 2009

A. (2 pts) Let $\vec{u} = \langle a, 4\sqrt{3} \rangle$ and $\vec{v} = \langle 1, \sqrt{3} \rangle$ form an angle of 60°. Solve for a.

ANSWER: _____ $a =$ _____

B. (3 pts) Given the diagram below, find the product of $\| 3\vec{v} - 2\vec{w} \|$ and $2\vec{u} \bullet \vec{v}$.



ANSWER: _____

C. (5 pts) A submarine dives from its dock at the origin and heads toward point (5 miles, 3 miles, $-\sqrt{2}$ miles) at a speed of 24 feet/second. Where is the submarine located after 10 seconds?

ANSWER: ⟨_____ feet, _____ feet, _____ feet⟩

Name_____ Score_____ School_____

Event 4: ANALYTIC GEOMETRY: Vectors in Two and Three Dimensions 2007

A. (2 pts) Given $\overrightarrow{AB} = 3\vec{i} + 5\vec{j}$ and $B(-1,-3)$, find A.

ANSWER A(_____, _____)

B. (3 pts) Given vectors $\vec{A}$ and $\vec{B}$ so that $|\vec{A}| = 8$, $|\vec{B}| = 5$ and the angle formed by $\vec{A}$ and $\vec{B}$ when placed tail to tail is 60°. Compute $|\vec{A} - \vec{B}|$.

ANSWER _____

C. (5 pts) $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \langle 3, 1, 2 \rangle$

Find the magnitude of $\mathbf{v} \times \mathbf{w}$.

ANSWER _____

Name_____ Score_____ School_____

Event 4: Algebra II - Vectors in Two and Three Dimensions 2005

A. (2 pts) Find $|\vec{r}|$ if $\vec{v} = (2, 5)$, $\vec{r} = (k, -3)$, and $\vec{v} \perp \vec{r}$.

ANSWER: _____

B. (3 pts) A divides $\overline{PQ}$ in a ratio of 2:3.
If $P = (-4, 16)$ and $Q = (20, 5)$, find A.

ANSWER: (_____ , _____)

C. (5 pts) If $\mathbf{v} = \left\langle -x, -1-x, \frac{4}{3}x \right\rangle$ and $\mathbf{w} = \left\langle 3x+3, \frac{4x-1}{2}, \frac{x^2+4}{2} \right\rangle$, what value of x makes $\mathbf{v}$ and $\mathbf{w}$ parallel?

ANSWER: _____

Name_____ Score_____ School_____

Event 5: Algebra II - Vectors in Two and Three Dimensions 2003

A. (2 pts) Given $\overrightarrow{AB} = 5\vec{i} + 7\vec{j}$ and B(–2, 3), find A.

ANSWER: (,)

B. (3 pts) Given $\vec{u} = \langle 2, 1 \rangle$ and $\vec{v} = \langle -1, -3 \rangle$.
Find the angle θ, $0° < \theta < 180°$, between the vectors.

ANSWER: °

C. (5 pts) The line through the points (1, 1, 2) and (3, 5, 8) intersects the plane $4x + 3y + 2z + 21 = 0$ at what point?

ANSWER: (, ,)

Name_____ Score_____ School_____

Event 5: Analytic Geometry - Vectors in Two and Three Dimensions 1999

A. (2 pts) Find the vector that is parallel and opposite to the vector $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$ and has a magnitude of 20 units.

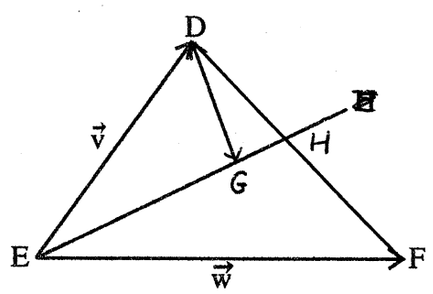
ANSWER: _____

B. (3 pts) Find all vectors $< -2, b, c >$ of length 3 that are perpendicular to $< 0, 2, -1 >$.

ANSWER: _____

C. (5 pts) In the figure below, given that $DH:HF = 2:3$ and $EG:GH = 2:1$. If $\vec{v} = \overrightarrow{ED}$ and $\vec{w} = \overrightarrow{EF}$, express $\overrightarrow{DG}$ in terms of $\vec{v}$ and $\vec{w}$.

D, H, F ARE COLLINEAR



ANSWER: _____

Name _____ Score _____ School _____

Event 3: EXTRA-CURRICULAR - Vectors 1997

A. (2 pts) Vector $\vec{u}$ has initial point (5, 2) and terminal point (-1, -8). Vector $\vec{v}$ has initial point (-7, 4) and terminal point (3, -2). Find $\vec{u}+\vec{v}$. Express your answer in the form $a\vec{i}+b\vec{j}$ where a and b are real numbers.

ANSWER _____

B. (3 pts) Determine the value of a so that $\vec{u}=3\vec{i}+a\vec{j}-2\vec{k}$ and $\vec{v}=2\vec{i}-4\vec{j}+7\vec{k}$ are perpendicular.

ANSWER _____

C. (5 pts) Find the equation of the plane containing the line $\frac{x-2}{3}=\frac{y+1}{-2}=z$ and the point $(0, 2, 5)$. Express your answer in the form $Ax+By+Cz=D$.

ANSWER _____

Meet 7, Event 4: ANALYTIC GEOMETRY
Vectors

2019

A. $\langle -8, 5, 9 \rangle$
B. $\sqrt{3}/3$
C. 11

2017

A. $\left\langle \frac{3\sqrt{2}}{10}, \frac{2\sqrt{2}}{5}, -\frac{\sqrt{2}}{2} \right\rangle$
B. $-2/3$
C. $\langle 3, 2, 6 \rangle$

2015

A. 3
B. 135°
C. 4

2013

A. $-15\vec{i} + 36\vec{j}$ or $\langle -15, 36 \rangle$
B. 135°
C. $\sqrt{10}$

2011

A. $\langle -3, 2 \rangle$, $\langle 3, -2 \rangle$
B. $(2, 3)$
C. $\frac{2}{3}\vec{u} + \frac{2}{3}\vec{v}$

2009

A. -4
B. $-60\sqrt{82}$
C. $\langle 200 \text{ feet}, 120 \text{ feet}, -40\sqrt{2} \text{ feet} \rangle$

2007

A. $(-4, -8)$
B. 7
C. $\sqrt{35}$

2005

A. $3\sqrt{29}/2$
B. $\left(\frac{28}{5}, \frac{58}{5} \right)$
C. -6

2003

A. $(-7, -4)$
B. 135°
C. $(-1, -3, -4)$

1999

A. $-12\hat{i} - 16\hat{j}$ or $-12\vec{i} - 16\vec{j}$ or $\langle -12, -16 \rangle$
B. $\langle -2, 1, 2 \rangle$, $\langle -2, -1, -2 \rangle$
C. $\frac{4}{15}\vec{w} - \frac{3}{5}\vec{v}$

1997

A. $4\vec{i} - 16\vec{j}$
B. -2
C. $13x + 17y - 5z = 9$

Note: Answers are shown as they appear on the original answer keys. There may be inconsistencies with the formatting of these answers. In all cases, consult the Guidelines for Forms of Answers to determine the correct formatting.