

Meet 7

EVENT 4: Calculus – Integrals (even-odd years)

- Include:
- (1) Computing definite and indefinite integrals
 - (2) Computing integrals by substitution
 - (3) Solving differential equations by separating variables
 - (4) Computing areas
 - (5) Computing volumes (using slices, disks, washers)
 - (6) Displacement & total distance traveled by a particle moving on a number line
 - (7) Average value
 - (8) The Fundamental Theorems of Calculus
 - (9) Using Riemann sums and trapezoids to estimate definite integrals
 - (10) Slope fields (no drawing)

- Exclude:
- (1) Integration by parts
 - (2) Trigonometric substitution
 - (3) Partial fractions
 - (4) Computing volumes using shells
 - (5) Computing lengths of curve, surface area, and centroids
 - (7) Trapezoidal error
 - (8) Simpson's Rule
 - (9) Parametric & polar functions
 - (1) Improper integrals

Sample Problems:

A-1. Compute: $\int \frac{x+1}{x} dx$
 Answer: $x + \ln|x| + C$

A-3. Compute: $\int_0^{\pi/2} \frac{\cos x dx}{\sqrt{1+3\sin x}}$
 Answer: $2/3$

A-2. Compute: $\int_0^{\ln 3} e^{2x} dx$
 Answer: 4

A-4. Compute: $\int_1^e \frac{\ln x}{x} dx$
 Answer: $1/2$

B-1. A particle moves on a number line with velocity $v(t) = 3t^2 - 12t + 9$. From $t = 0$ to $t = 5$, find the total distance traveled by the particle.
 Answer: 28

B-2. Find the area of the region bounded by $y = \sqrt{x}$ and $y = \frac{x^2}{8}$.
 Answer: $8/3$

B-3. Water is flowing into a tank at a rate of $\frac{t}{t^2 + 1}$ gallons per minute (t measured in minutes). If the tank is empty to begin with, how many gallons will it contain at the end of 10 minutes?
 Answer: $\frac{\ln 101}{2}$

B-4. Solve the differential equation $\frac{dy}{dx} = \frac{2x}{y^2}$ with initial condition $y(1) = 3$.
 Answer: $y = \sqrt[3]{3x^2 + 24}$

B-5. f is a continuous function with values as shown below. Estimate

$\int_{-1}^{17} f(x) dx$ using the midpoint of 3 rectangles.

x	-1	2	5	8	11	14	17
$f(x)$	1	2	3	5	8	13	21

Answer: 120

B-6. The velocity (v) of a particle moving on a number line is a continuous function with values as shown below. Estimate the distance traveled from $t = 1$ to $t = 10$ using 3 trapezoids.

t	1	3	6	10
$v(t)$	7	5	3	2

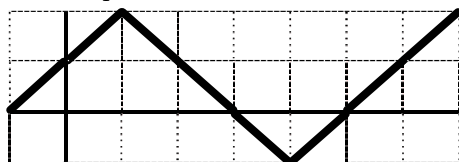
Answer: 34

B-7. Compute the average value of $f(x) = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{2}$.

Answer: $\frac{2}{\pi}$

B-8. Given the graph of $y = f(x)$ defined on $-1 \leq x \leq 7$ as shown below, and

$g(x) = \int_1^x f(t) dt$. Compute $g(-1)$.



Answer: -2

B-9. The base of a solid is the region bounded by $y = \sqrt{x}$, $y = 2$ and the y -axis. Compute the volume of the solid if cross sections perpendicular to the y -axis are squares.

Answer: $\frac{32}{5}$

C-1. R is the region bounded by $y = \sqrt{x}$, $y = 2$ and the y -axis. Compute volume of the solid generated when R is revolved about the line $y = 2$.

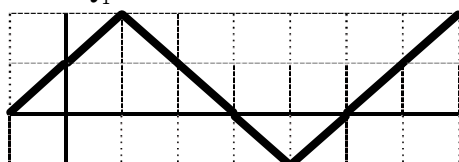
Answer: $\frac{8\pi}{3}$

C-2. The region bounded by $y = 1/x^2$, $y = 0$, $x = 1$ and $x = 4$ is divided into 2 regions of equal area by the vertical line $x = k$. Find the value of k .

Answer: $\frac{8}{5}$

C-3. Given the graph of $y = f(x)$ defined on $-1 \leq x \leq 7$ as shown below, and

$g(x) = \int_1^x f(t) dt$. At what value(s) of x does g have a point of inflection?



Answer: 1, 4

- A. Find the average value of $f(x) = 3\sqrt{x+1}$ over the interval $[-1, 8]$.
(2 pts)

ANSWER: _____

- B. Let R be the region in the first quadrant bounded by the graph of $y = \sqrt{\cos(4x)}$, the x -axis, and the y -axis. The region R is the base of a solid. For this solid, the cross-sections perpendicular to the x -axis are equilateral triangles with one side in the region R. Find the volume of this solid.
(3 pts)

ANSWER: _____

- C. Find the area of the region enclosed by $x - y^2 = 0$ and $x + 2y^2 = 3$.
(5 pts)

ANSWER: _____

A. Evaluate: $\int_0^{\frac{\pi}{2}} e^{\sin x} \cos x \, dx$

(2 pts)

ANSWER: _____

B. Find the average value of the function $f(x) = 8x - 3 + 5e^{2-x}$ from $x = 1$ to $x = 4$.

(3 pts)

ANSWER: _____

C. Region R is bounded by $3x - y + 1 = 0$ and $y = x^2 + 1$. Find the volume of the solid generated by revolving R about the y -axis.

(5 pts)

ANSWER: _____

Meet 7, Event 4: CALCULUS
Integrals

2019

- A. 6
- B. $\sqrt{3}/16$
- C. 4

2018

- A. $e - 1$
- B. $17 - \frac{5}{3e^2} + \frac{5e}{3}$
- C. $27\pi/2$