Meet 7

EVENT 4: Analytic Geometry – Rational Functions (Odd Years)

Notes: (1) The x-intercept is the first coordinate (the number) of the point(s) of intersection of the graph with the x-axis. The y-intercept is the second coordinate of the point(s) of intersection of the graph with the y-axis. If the problem calls for the x- and y-intercepts, to avoid ambiguity, the form of the answer blank should be:

Answer: x-intercept(s) _____
y-intercept(s) _____

and the appropriate number(s) should be listed. If there is no x-intercept and/or y-intercepts, write "none".

(2) Asymptotes are lines and should be described by the appropriate linear equation (first degree in x and/or y).

(3) Set or interval notation acceptable for domain and range.

Include: (1) x-intercepts and y-intercepts
(2) Vertical, horizontal, and slant asymptotes
(3) Domain and range of the function
(4) Symmetry with respect to the y-axis and origin

Sample Problems:

A. Find the x-intercept(s) for $y = \dfrac{x^2 - 4}{x^2 - 25}$.

Answer: $2, -2$

B. Find the asymptotes for $y = \dfrac{x^3}{x^2 + x - 12}$.

Answer: vertical: $x = 3,\ x = -4$
horizontal: none
slant: $y = x - 1$

C. Determine the domain and range of $\left\{(x, y) : y = \dfrac{4x^2 + 1}{x^2 - 1}\right\}$.

Answer: domain: $\{x : x \neq \pm 1\}$
range: $\{y : y \leq -1 \text{ or } y > 4\}$

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Rational Functions

April 2018

A. Find the y-intercept of the graph of $y = \dfrac{(x^2 - 36)(2x^2 - 8)}{x^2 + 8x + 12}$.

(2 pts)

ANSWER: _____

B. Find the point of intersection of the asymptotes of the graph of $y = \dfrac{x^3 + 8x^2 + 5x - 50}{x^2 - 4x + 4}$.

(3 pts)

ANSWER: _____ (,) _____

C. Find the domain and the range of $\left\{(x,y) : y = \dfrac{x^2}{4 - x^2}\right\}$.

(5 pts)

ANSWER: domain: {x: _____ }

range: {y: _____ }

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Rational Functions

April 2016

A. (2 pts) Find the x- and y-intercept(s) of $-5xy + x^2 - 3x + 4y - 5 = 0$.

x - intercept(s): _____

ANSWER: y - intercept(s): _____

B. (3 pts) Find the zeros of $f(x) = \dfrac{x^3 - 2x^2 - 5x + 6}{x^3 + 3x^2 + 2x}$.

ANSWER: _____

C. (5 pts) Find the area of the triangle in the first quadrant bounded by the x-axis and the slant and vertical asymptotes of $f(x) = \dfrac{x^3 - 12x^2 - x + 15}{3 - 3x^2}$.

ANSWER: _____

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Rational Functions

April 2014

A. Find the zeros of $y = \dfrac{(x+3)(x^2-4)}{(x+2)(x^2-5)}$.

(2 pts)

ANSWER: _____

B. Find all asymptotes of $y = \dfrac{x^2+5}{x-2}$. If there is no asymptote of a particular type, write "none".

(3 pts)

ANSWER: Horizontal: _____

Vertical: _____

Slant: _____

C. At what point does the graph of $y = \dfrac{(x-1)(2x^3+3x-5)}{x^3-x^2+2x-2}$ cross its asymptote?

(5 pts)

ANSWER: _____ (_____ , _____) _____

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Rational Functions 2012

A. (2 pts) In a certain national park, the deer population is estimated by $N = \dfrac{10(5+3t)}{1+0.04t}$, where N is the number of deer and t is the time in years, $t \geq 0$. What is the limiting size of the herd as time increases?

ANSWER: _____deer_____

B. (3 pts) Find all points of intersection of the asymptotes of $y = \dfrac{3x^3 - 10x^2 - 27x + 10}{x^2 - 3x - 10}$.

ANSWER: _____

C. (5 pts) A certain rational function $R(x)$ can be written as $\dfrac{p(x)}{q(x)}$, where $p(x)$ is a polynomial of degree 2. $R(x)$ has vertical asymptote $x = -3$ and slant asymptote $y = -2x + 5$. If 1 is a zero of $R(x)$, find $R(x)$.

ANSWER: $R(x) =$ _____

Name _____ Score _____ School _____

Event 4: ANALYTIC GEOMETRY — Rational Functions 2010

A. (2 pts) Find the x- and y-intercept(s) of the following: $y = \dfrac{4x^2 - 4}{x^4 - 16}$

ANSWER: x-intercept(s): _____

y-intercept(s): _____

B. (3 pts) At what points does the graph of $y = \dfrac{8x^3 + 13x^2 + 21x + 26}{2x^3 + 3x^2 + 5x + 7}$ cross its horizontal asymptote?

ANSWER: _____

C. (5 pts) Determine the range: $x^2y - x^2 - 4y + 1 = 0$

ANSWER: { y: _____ }

Name_____ Score_____ School_____

Event 4: ANALYTICAL GEOMETRY – Rational Functions 2008

A. (2 pts) Let $\varphi(x) = \dfrac{20x^2 - 11}{\dfrac{x}{x+2} - \dfrac{x}{x-2}}$. Find the domain of φ.

ANSWER $\{x:$ _____ $\}$

B. (3 pts) Find all asymptotes for the graph of:

$-x^3 + x^2 + x^2y + x + 2xy - 3y - 1 = 0$.

ANSWER Horizontal _____

Vertical _____

Slant _____

C. (5 pts) Let $f(x) = \dfrac{x+1}{x+2}$, $g(x) = \dfrac{x-3}{x-2}$ and $h(x) = (f \circ g)(x)$. Find the domain and range of h.

ANSWER Domain: $\{x:$ _____ $\}$

Range: $\{y:$ _____ $\}$

Name_____ Score_____ School_____

Event 4: Analytic Geometry - Rational Functions 2006

A. (2 pts) Find the x-intercepts and y-intercepts of

$$f(x)=\frac{5x^2-7x}{2x^2-50}.$$

ANSWER: x-int. = _____

y-int. = _____

B. (3 pts) Find all the asymptotes of the function $f(x)=\frac{x^3+2x^2-1}{2x^2-5}$. If there is no asymptote of a particular type, write "none."

ANSWER: Horizontal: _____

Vertical: _____

Slant: _____

C. (5 pts) A certain rational function $R(x)$ can be written as $\frac{p(x)}{q(x)}$ where $p(x)$ is a polynomial of degree 2. $R(x)$ has vertical asymptote $x=2$ and slant asymptote $y=x+2$. 3 is a zero of $R(x)$. Find $R(x)$.

ANSWER: $R(x)$ = _____

Name_____ Score_____ School_____

Event 5: Algebra II - ~~Vectors in Two and Three Dimensions~~ RATIONAL FUNCTIONS 2004

A. (2 pts) Find the x-intercepts of:

$$f(x) = \frac{-x^2 - x + 6}{x^2 + 3x - 4}$$

ANSWER: x-intercept(s) _____

B. (3 pts) Find all points of intersection of the asymptotes of:

$$y = \frac{3x^3 - 2x^2 + x + 1}{x^2 - 4x + 3}$$

ANSWER: _____

C. (5 pts) Find the domain and range for:

$$y = \frac{x^3 - 4x}{x^3 - 9x}$$

ANSWER: Domain = $\{x:$ _____ $\}$

Range = $\{y:$ _____ $\}$

Name_____ Score_____ School_____

Event 5: Algebra II - Rational Functions 2002

A. (2 pts) Find the x and y-intercepts of

$$y = \frac{3x-2}{x^2-4}$$

ANSWER: x-intercept(s): _____

y-intercept(s): _____

B. (3 pts) Given the rational function $y = \frac{Ax^2+Bx+C}{x^2+D}$.

Find the values of A, B, C, and D so that the function is even, has asymptotes $x = 2$ and $y = 3$, and contains the point $(1, 2)$.

ANSWER: $A =$ _____ $B =$ _____ $C =$ _____ $D =$ _____

C. (5 pts) At what point does the graph of $y = \frac{2x^3+3x^2+5x+7}{x^2+2x+3}$ cross its slant asymptote?

ANSWER: (_____ , _____)

Name _____ Score _____ School _____

Event 5: ANALYTIC GEOMETRY - Rational Functions 2000

A. (2 pts) Determine whether the graph of the following is symmetric with respect to the x-axis, the y-axis, and/or the origin. If so, place a "yes" in the appropriate answer blank. If not, answer "no."

$$y = \frac{x^3 - x}{x^2 + 5}$$

ANSWER x-axis: _____
y-axis: _____
origin: _____

B. (3 pts) Find all asymptotes for the graph of

$$2x^3 - x^2y + 2x^2 - 3xy - 12x + 10y = 0$$

If there is no asymptote of a particular type, write "none."

ANSWER Vertical: _____
Horizontal: _____
Slant: _____

C. (5 pts) Determine the range:

$$x^2y - xy - 6y - 4 = 0$$

ANSWER Range: $\{y \mid$ _____ $\}$

Name _____ Score _____ School _____

Event 5: ANALYTIC GEOMETRY – Rational Functions

1998

A. (2 pts) Find the x- and y-intercept(s) for the graph of:

$$2x^2 + 2xy - x - y - 2 = 0$$

ANSWER x-intercept(s): _____

y-intercept(s): _____

B. (3 pts) Find the equation(s) of all asymptotes for the graph of: $F(x) = \frac{f(x)}{g(x)}$

where $f(x) = x^3 - 1$ and $g(x) = x^2 - 1$. If there is no asymptote of a particular kind, write "None" in the corresponding answer space.

ANSWER: Vertical: _____

Horizontal: _____

Slant: _____

C. (5 pts) Find the domain:

$$f(x) = \frac{\sqrt{x^2 + 5x + 4}}{\sqrt{x^2 - 3x - 10}}$$

ANSWER Domain: {x: _____}

Name _____ Score _____ School _____

Event 5: ANALYTIC GREOMETRY - Algebraic curves in the Çartesian plane 1997

A. (2 pts) Find the x- and y-intercepts of $y = \dfrac{(x^2-36)(2x^2-8)}{x^2+8x+12}$

ANSWER x-intercept(s): _____

y-intercept(s): _____

B. (3 pts) Determine the equation of each asymptote for the graph of $x^3+36x^2-3x^2y-18x+18y+20=0$. If the graph does not have an asymptote of a certain type, write "none".

ANSWER horizontal: _____

slant: _____

vertical: _____

C. (5 pts) Find the domain and range of $\{(x,y): 4x^2+8x+9y^2-36y+4=0\}$.

ANSWER domain: { x: _____ }

range: { y: _____ }

Name _____ Score _____ School _____

Event 5: ANALYTIC GEOMETRY - Algebraic curves in the Cartesian plane 1996

A. (2 pts) Find the x-intercept(s) of the graph of $2x^3 - 2x^2 - x - y = 0$

ANSWER x-intercept(s): _____

B. (3 pts) Find the range of $y = x + \frac{1}{x}$.

ANSWER {y: _____ }

C. (5 pts) Determine a, b, c, and d, given that the graph of $y = \frac{ax^2 + bx + c}{x + d}$ has a slant asymptote $y = 5x - 3$, vertical asymptote $x = -4$, and an x-intercept of -2.

ANSWER $a =$ _____, $b =$ _____, $c =$ _____, $d =$ _____

Name _____ Score _____ School _____

Event 5: ANALYTIC GEOMETRY - Algebraic Curves in the Cartesian Plane 1995

A. (2 pts) Find the zero(s) of:

$$y = \frac{(x - 2)^2 (x^2 + 2x - 35)}{(x^2 - 7x + 10)(x - 2)}$$

ANSWER _____

B. (3 pts) Find all asymptotes of the graph of:

$$y = \frac{2x^3 + 5x^2 - 3x}{x^2 + x - 6}$$

If there is no asymptote of a particular kind, write "None" in the corresponding blank.

ANSWER: Vertical: _____

Horizontal: _____

Slant: _____

C. (5 pts) Find the domain and range of the relation with equation

$$12x^2 + y^2 + 13 = 4y - 24x$$

ANSWER Domain: {x: _____}

Range: {y: _____}

Meet 7, Event 4: ANALYTIC GEOMETRY
Rational Functions

2018

A. 24
B. $(2, 14)$
C. domain: $\{x : x \neq \pm 2\}$
range: $\{y : y \geq 0 \text{ or } y < -1\}$

2016

A. x-intercepts: $\frac{3 \pm \sqrt{29}}{2}$
y-intercept: $5/4$
B. $1, 3$
C. $121/6$

2014

A. $-3, 2$
B. Horizontal: none
Vertical: $x = 2$
Slant: $y = x + 2$
C. $(-5, -10)$

2012

A. 750
B. no points of intersection
C. $\frac{-2x^2 - x + 3}{x + 3}$

2010

A. x-intercept(s): ± 1
y-intercept(s): $1/4$
B. $(-2, 4), (1, 4)$
C. $\{y : y > 1 \text{ or } y \leq 1/4\}$

2008

A. $\{x \mid x \neq \pm 2 \text{ and } x \neq 0\}$
B. Horizontal: none
Vertical: $x = -3$
Slant: $y = x - 3$
C. $\{x : x \neq 7/3, x \neq 2\}$
$\{y : y \neq 2/3, y \neq 1\}$

2006

A. x-intercept: $0, 7/5$
y-intercept: 0
B. Horizontal: none
Vertical: $x = \pm\sqrt{10}/2$
Slant: $y = \frac{1}{2}x + 1$
C. $y = \frac{x^2 - 9}{x - 2}$

2004

A. x-intercept: $-3, 2$
B. $(3, 19), (1, 13)$
C. Domain: $\{x : x \neq 0 \text{ and } x \neq \pm 3\}$
Range: $\{y : y < 4/9 \text{ or } y > 1\}$

2002

A. x-intercept: $2/3$
y-intercept: $1/2$
B. $A = 3, B = 0, C = -9, D = -4$
C. $(-10, -21)$

2000

A. x-axis: No
y-axis: No
Origin: Yes
B. Vertical: $x = -5$
Horizontal: none
Slant: $y = 2x - 4$
C. $\{y \mid y > 0 \text{ or } y \leq -16/25\}$

1998

A. x-intercept: $\frac{1 \pm \sqrt{17}}{4}$
y-intercept: -2
B. Vertical: $x = -1$
Horizontal: none
Slant: $y = x$
C. Domain: $\{x \mid x \neq -2 \text{ and } x \neq 1\}$
Range: $\{y \mid y \geq -9/4\}$

1997

A. x-intercept: $2, 6$
y-intercept: 24
B. Vertical: $x = \sqrt{6}, x = -\sqrt{6}$
Horizontal: none
Slant: $y = \frac{1}{3}x + 12$
C. Domain: $\{x : -4 \leq x \leq 2\}$
Range: $\{y : 0 \leq y \leq 4\}$

1996

A. $0, \frac{1 + \sqrt{3}}{2}, \frac{1 - \sqrt{3}}{2}$
B. $\{y : y \leq -2 \text{ or } y \geq 2\}$ or $\{y : |y| \geq 2\}$
C. $a = 5, b = 17, c = 14, d = 4$

Note: Answers are shown as they appear on the original answer keys. There may be inconsistencies with the formatting of these answers. In all cases, consult the Guidelines for Forms of Answers to determine the correct formatting.

1995

A. -7

B. Vertical: $x = 2$
Horizontal: none
Slant: $y = 2x + 3$

C. Domain: $\left\{x : -\frac{3}{2} \le x \le -\frac{1}{2}\right\}$

Range: $\left\{y : 2 - \sqrt{3} \le y \le 2 + \sqrt{3}\right\}$

Note: Answers are shown as they appear on the original answer keys. There may be inconsistencies with the formatting of these answers. In all cases, consult the Guidelines for Forms of Answers to determine the correct formatting.