

## Meet 7

### EVENT 6: Calculus – Integrals

- Include:
- (1) Computing definite and indefinite integrals
  - (2) Computing integrals by substitution
  - (3) Solving differential equations by separating variables
  - (4) Computing areas
  - (5) Computing volumes (using slices, disks, washers)
  - (6) Displacement & total distance traveled by a particle moving on a number line
  - (7) Average value
  - (8) The Fundamental Theorems of Calculus
  - (9) Using Riemann sums and trapezoids to estimate definite integrals
  - (10) Slope fields (no drawing)

- Exclude:
- (1) Integration by parts
  - (2) Trigonometric substitution
  - (3) Partial fractions
  - (4) Computing volumes using shells
  - (5) Computing lengths of curve, surface area, and centroids
  - (7) Trapezoidal error
  - (8) Simpson's Rule
  - (9) Parametric & polar functions
  - (1) Improper integrals

#### Sample Problems:

A-1. Compute:  $\int \frac{x+1}{x} dx$   
 Answer:  $x + \ln|x| + C$

A-3. Compute:  $\int_0^{\pi/2} \frac{\cos x dx}{\sqrt{1+3\sin x}}$   
 Answer:  $2/3$

A-2. Compute:  $\int_0^{\ln 3} e^{2x} dx$   
 Answer:  $4$

A-4. Compute:  $\int_1^e \frac{\ln x}{x} dx$   
 Answer:  $1/2$

B-1. A particle moves on a number line with velocity  $v(t) = 3t^2 - 12t + 9$ . From  $t = 0$  to  $t = 5$ , find the total distance traveled by the particle.  
 Answer:  $28$

B-2. Find the area of the region bounded by  $y = \sqrt{x}$  and  $y = \frac{x^2}{8}$ .  
 Answer:  $8/3$

B-3. Water is flowing into a tank at a rate of  $\frac{t}{t^2+1}$  gallons per minute ( $t$  measured in minutes). If the tank is empty to begin with, how many gallons will it contain at the end of 10 minutes?  
 Answer:  $\frac{\ln 101}{2}$

B-4. Solve the differential equation  $\frac{dy}{dx} = \frac{2x}{y^2}$  with initial condition  $y(1) = 3$ .  
 Answer:  $y = \sqrt[3]{3x^2 + 24}$

B-5.  $f$  is a continuous function with values as shown below. Estimate

$\int_{-1}^{17} f(x) dx$  using the midpoint of 3 rectangles.

$x$	-1	2	5	8	11	14	17
$f(x)$	1	2	3	5	8	13	21

Answer: 120

B-6. The velocity ( $v$ ) of a particle moving on a number line is a continuous function with values as shown below. Estimate the distance traveled from  $t = 1$  to  $t = 10$  using 3 trapezoids.

$t$	1	3	6	10
$v(t)$	7	5	3	2

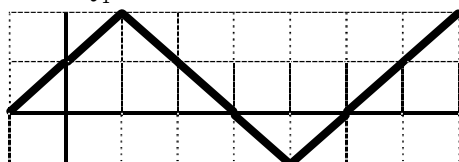
Answer: 34

B-7. Compute the average value of  $f(x) = \sin 2x$  from  $x = 0$  to  $x = \frac{\pi}{2}$ .

Answer:  $\frac{2}{\pi}$

B-8. Given the graph of  $y = f(x)$  defined on  $-1 \leq x \leq 7$  as shown below, and

$g(x) = \int_1^x f(t) dt$ . Compute  $g(-1)$ .



Answer: -2

B-9. The base of a solid is the region bounded by  $y = \sqrt{x}$ ,  $y = 2$  and the  $y$ -axis. Compute the volume of the solid if cross sections perpendicular to the  $y$ -axis are squares.

Answer:  $\frac{32}{5}$

C-1.  $R$  is the region bounded by  $y = \sqrt{x}$ ,  $y = 2$  and the  $y$ -axis. Compute volume of the solid generated when  $R$  is revolved about the line  $y = 2$ .

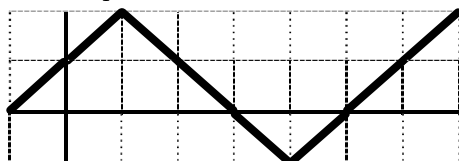
Answer:  $\frac{8\pi}{3}$

C-2. The region bounded by  $y = 1/x^2$ ,  $y = 0$ ,  $x = 1$  and  $x = 4$  is divided into 2 regions of equal area by the vertical line  $x = k$ . Find the value of  $k$ .

Answer:  $\frac{8}{5}$

C-3. Given the graph of  $y = f(x)$  defined on  $-1 \leq x \leq 7$  as shown below, and

$g(x) = \int_1^x f(t) dt$ . At what value(s) of  $x$  does  $g$  have a point of inflection?



Answer: 1, 4

A. Evaluate:  $\int_0^{\frac{\pi}{2}} e^{\sin x} \cos x \, dx$   
(2 pts)

ANSWER: \_\_\_\_\_

B. Find the average value of the function  $f(x) = 8x - 3 + 5e^{2-x}$  from  $x = 1$  to  $x = 4$ .  
(3 pts)

ANSWER: \_\_\_\_\_

C. Region R is bounded by  $3x - y + 1 = 0$  and  $y = x^2 + 1$ . Find the volume of the solid generated by revolving R about the  $y$ -axis.  
(5 pts)

ANSWER: \_\_\_\_\_

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A.  $e - 1$

B.  $17 - \frac{5}{3e^2} + \frac{5e}{3}$

C.  $27\pi/2$

Note: Answers are shown as they appear on the original answer keys. There may be inconsistencies with the formatting of these answers. In all cases, consult the Guidelines for Forms of Answers to determine the correct formatting.